A COMPENDIUM

OF THE

SCIENCE OF LOGIC

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PREFACE

In writing a Compendium of Logic, I had this purpose in view: while leaving ample room for the professor to develop the matter in his own way, I intended to induce the student to think for himself. The tendency to learn merely by heart is a habit which in Logic as in other rigorous sciences becomes absurd.

In fact, scientific learning demands a grasp of the subject as a whole, a comprehension of its various sections, and a reason to account for each conclusion as related to the systematic body of doctrine. Such an ideal aim of learning postulates in the student a constant application to study, an effort to understand, in the light of explanations received, the full meaning of definitions and topics as presented throughout the course of the text-book.

It is worth noticing that this endeavour—apart from the knowledge obtained—brings about the intellectual pleasure of adding to each lesson something which the student considers as his own, increases the power of thinking and creates a sense of confidence in knowledge gained by personal industry.

The Questions and Exercises will contribute to the end proposed, stimulating the mind to work out exact solutions. Moreover, they will afford a test as to what the student can do, when obliged to tackle unexpected problems.

ST. XAVIER'S COLLEGE,
BOMBAY, *December*, 1933

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INTRODUCTION

1. How to Study a Science. Whoever enters for the first time into a new field of knowledge should acquaint himself with the elements of that science arranged in an orderly epitome. A didactic method of study is the surest way to progress.

“ Some people,” says Balmes, “ wish to be excused from it on the ground that articles in dictionaries contain enough to enable a man to speak of everything while understanding nothing ; but reason and experience tell us that such a method can only produce superficial learning.

“ In fact, there is in every science and profession an aggregate of primordial notions, names and expressions which can only be learned by studying an elementary work ; so that even though there were no other considerations, the present one would be sufficient to demonstrate the drawbacks of taking any other road. Those primordial notions, names and expressions, should be treated with respect by him who enters for the first time on any new study. He should take for granted that those who preceded him in this study, and invented those words, knew what they were doing. If the freshman distrusts his predecessors, if he hopes to be able to reform the science or profession, or perhaps radically change it, he should at least reflect that it is prudent to see what others have said ; that it is rash to try to create everything himself; and that by refusing to avail himself of his neighbour's labours, he exposes himself to losing a great deal of time.

"The advice given to those who commence the study of history is applicable to all classes of students: before you begin its study read a compendium. We may here quote the remarkable words of Bossuet in the dedicatory to his *Discourse on Universal History*. He speaks of the necessity of studying history in a compendium, to avoid confusion and save trouble, and then he adds: 'We may compare this manner of exposing to view universal history, to the description of geographical maps: universal history is the general map, compared with the partial maps of each country and province. In the partial maps you see minutely what a kingdom or a province is in itself: in the universal maps you learn to connect these parts of the world with the whole: in a word, you see the part which Paris occupies in the kingdom of France, what the kingdom occupies in Europe and Europe in the universe.' Very well: the luminous comparison between the map of the world and partial maps holds in all branches of knowledge. In all there is an aggregate with which we must become acquainted, in order to comprehend the parts and to avoid confusion in arranging them. Ideas which are acquired without method are almost always incomplete, often inexact, and sometimes false; but all these drawbacks are as nothing, compared with those which result from undertaking in the dark, without antecedent or guide, the study of a science.

"Between those who have studied a science in its principles, and those who have gathered some slight notions of it from encyclopædias and dictionaries, there is always a marked difference which does not escape the experienced eye. The former are distinguished for precision of ideas and propriety of language; the latter may display abundant and select knowledge, but at a moment least expected they make an egregious stumble which manifests their ignorant superficiality." [1]

2. Difficulties in the Course of Logic. The above considerations are particularly pressing on those who begin the study of Logic.

"Logic," says Devey, "owing to the opposite schools of metaphysicians, has met with various and even conflicting treatment in nearly every age. It appears to be the only subject of which the difficulties multiply with the spread of knowledge, and which, instead of disentangling itself from the quarrels of the past, periodically carries forward all its old perplexities to be added to the account of the present." [2]

To make the point concrete, the following stumbling-blocks are not uncommon in modern books of Logic:—(1) It is not clearly defined what a concept is, and whether a concept refers to reality or has a mere subjective significance. As a consequence of this confusion, logical truth is perverted by calling it consistency of the mind with itself; (2) An attempt has been made to represent the act of judgment by a sign of equality between two concepts, which kills to the root the life of a proposition; (3) The syllogism as a right form of inference has been questioned; (4) The uniformity of nature as a formal ground of Induction is, according to some logicians, a supposition or at most a subjective belief.

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[1] See J. Balmes, *El Criterio*, Chap. XVII. The work is known in English as *The Art of Thinking Well*, a translation by Rev. W. McDonald, D.D.

[2] Joseph Devey, *Logic*, Preface.

Views of this kind, imported from systems of Philosophy into the field of Logic, are apt to bewilder the student treading on the road of correct thinking.

"Now, this," we say with Devey, "is not a satisfactory state of that science which concerns the operation of the faculty in which men chiefly glory, not only as the mark which pre-eminently divides them from the brute creation, but also as the primary source of those distinctions of rank and supremacy which obtain in society. Nor has the writer met with anything in logical treatises to necessitate the conflicting diversity of view in which they regard the science. There is nothing in the body of Aristotle's speculative views to hinder them from being engrafted on the practical doctrines of Bacon; nor anything in the *a priori* methods of Descartes essentially antagonistic even to the Inductive methods propounded by Comte and Helvetius. Apart from the metaphysical tenets of these schools, the general body of their logical doctrines may be combined in one system." [1]

The right thing to do is to distinguish and keep the principles of Logic apart from the theories by which those principles are established. It is gratifying to observe that, when our science is maintained within its proper boundaries, the traditional logical principles are better fixed than those of other sciences. They being based on the natural process of thinking, and having passed through the test of many centuries, are not liable to change with the theories of knowledge. What may change, and actually does change, is the manner of their application to the problems of ever-growing human knowledge.

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[1] J. Devey, *Logic*, Preface.

To use a comparison, one may know very well the road that will carry him from Bombay to Poona, without knowing how the road was built, and a thousand other questions. These in fact will not interest him or make the journey easier. So it is with the acts of simple apprehension, judgment and reasoning. If I think, for instance, of a motor-car and judge that I can buy it, and reason that much time can be saved by actually buying it, I am certain of my business without discussing the validity of ideas or studying the many theories about the bridge between the mind and reality.

It is of the utmost importance to uphold the soundness of our science ; for if Logic has to guide correct thinking, it must be correct in itself. A shaky rule cannot be accepted as the standard of rectitude.

3. **Our Exposition of Logic.** An attempt has been made to put before the fresh student the elements of the science of Logic in a concise manner, yet complete as to contents, and up-to-date in illustration. The purpose is to place him in a position to follow the explanations of a professor, or to understand the matter as presented in the standard books. Regarding controverted points the prevailing opinion is given, which may be considered as the mean between extremes. Discussions are purposely avoided, but reference is made to conflicting authorities.

We realise that theory is not enough. Let us remember that the end of teaching is twofold ; first to impart knowledge and secondly to develop the mind, so that the pupil on leaving school may be able to proceed on his own initiative ; hence the necessity of practical exercises. In this regard hints and a few examples are given along with the text to illustrate the doctrine and its application. But a good deal of questions and exercises arranged by the order of chapters will be found at the end. Practical work is strictly individual and must be carried out by each one under the guidance of a tutor.

As to the extension of the field of Logic, it is generally admitted nowadays that it ought to embrace the process of Induction as well as that of Deduction. Sir W. Hamilton [1] looked at Logic from the point of Deductive reasoning, while J. S. Mill [2] took his view from Inductive methods. The modern writers, J. Welton,[3] P. Coffey,[4] Bosanquet,[5] Johnson,[6] and others who treat of the science in full take equal account of both sections.

Some difference exists, however, as to the manner of combining them. Welton writes : " Logic is an organic whole. This accounts for the order in which the topics are treated. The customary arrangement is due partly to historical reasons, partly to pedagogical theory. Induction is made to follow Deduction because it was added in the last century to the traditional treatment. But this does not represent its place in the actual work of thought." [7]

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[1] Sir William Hamilton, *Lectures on Logic*, Vols. I., II., 1860.

[2] John Stuart Mill, *A System of Logic*, Vols. I., II., 1868.

[3] J. Welton, *A Manual of Logic* Vols. I., II., 1912.

[4] P. Coffey, *The Science of Logic*, Vols. I, II., 1918.

[5] Bernard Bosanquet, *Logic, or the Morphology of Knowledge*, Vols. I., II., 1911.

[6] W. E. Johnson, *Logic*, Parts I., II., III., (IV.), 1921.

[7] J. Welton, *Groundwork of Logic*, Preface, 1920.

On mere didactic grounds I consider it better to treat of the Inductive process after the principles of Deduction, not as a sequel, but as a counterpart of them. Induction shows the organic constitution of those principles. The reasons are analogous to those for teaching plane Geometry before solid Geometry. To put it briefly, it is easier to place ourselves on the top of a mountain and descend, than to climb up-hill, though apparently we cover the same ground.

PART I

THE PRINCIPLES AND PROCESSES OF DEDUCTIVE REASONING

CHAPTER I

GENERAL SURVEY OF LOGIC

1. Natural Origin of Logic. The science and art of Logic, as any other branch of knowledge, came out of the study of nature, which, like a book, lies open before us. Its particular province is the activity of our own mind. Granted that man thinks with the power of reflecting upon the operations of his own reason, it was natural—and required no great effort—to find out that human reasonings are at times correct and at other times wrong. Such experience, together with innate curiosity, gave rise to an inquiry into the conditions of correct thinking.

Who was the pioneer enquirer into those conditions, or who it was that first laid down the principles of correct thinking, is not precisely known. Some kind of Logic may be traced in the most ancient currents of thought, namely, the Chinese, the Indian and the Greek philosophies.

2. Historical Sketch. The system of Logic that we possess is derived from the classical period of the Greek philosophy, and is invariably associated with one name, Aristotle, the greatest philosopher of antiquity and founder of the syllogism.

Socrates (469-400 B.C.), led by a sincere love of truth, started the use of definitions as a means to correct thinking. He obtained definitions by a kind of induction of his own, commonly called "The Socratic Method," which consists in drawing out people's ideas by a series of questions, and if their ideas are wrong, making them correct themselves by answering further questions. Thus determining the strict meaning of terms and their definitions, he opposed and unmasked the prevailing school of the Sophists, whose artful scheme consisted in playing with ambiguity in the course of argument.

Plato (429-347 B.C.), a pupil and ardent admirer of Socrates, advanced much further in the same direction. He explained the meaning of universal ideas, and how universality is the foundation of science. He extended the use of definitions, and established a distinction among the various parts of speech. To this he added the analysis of terms, and the use of division as a means to set in order our complex ideas of classes of things.

Aristotle (384-322 B.C.) learned for twenty years the doctrine of Plato as a pupil, and as a master carried on the precepts of Logic to a marvellous degree of perfection. We may remark in passing that for him universal concepts were not an intuition of pre-existing platonic ideas, but notions obtained from things by mental abstraction. This sound doctrine has been maintained by the Schoolmen and their followers to this day. He wrote clearly on Names and their relations, on Definition, on Division, on Propositions, on the Syllogism—which was altogether an invention of his own; on Probable arguments and on Fallacies. All these treatises, accurate, concise and without repetitions, are, as they came from the pen of Aristotle, a treasure of logical principles handed down to us for our respect and admiration.

These treatises, on account of their utility, were grouped by his followers under one title, the Organon of Aristotle, towards the beginning of the Christian Era. The name Logic became current among the Stoics. Porphyry (A.D. 232-304), a Greek philosopher of the Neoplatonic School, wrote a valuable commentary on Aristotle's Categories and Predicables.

Boethius (A.D. 470-525), made known the doctrines of Plato and Aristotle to the schools of the Roman Empire by a praiseworthy translation into Latin of the works of these two great leaders of thought. The Schoolmen of the Middle Ages, particularly Albert the Great (1193-1280) and Thomas Aquinas (1225-1274), applied extensively and commented upon the Logic of Aristotle, which they adopted as their own.

Lord Francis Bacon (1561-1626) in his *Novum Organum*, laid stress on a new inductive Art leading to scientific truth, namely, the interpretation of nature. " The Organon of Aristotle and the Organum of Bacon," says Hamilton, " stand in relation, but the relation of contrariety : the one considers the laws under which *the subject* thinks ; the other, the laws under which *the object* is to be known. To compare them together is therefore, in reality, to compare together quantities of different species. Each proposes a different end ; both in different ways are useful, and both ought to be assiduously studied." [1]

Isaac Newton (1642-1727) was a true pioneer of inductive logic, particularly for deciding against Descartes that natural causes are discovered not by *a priori* reasoning but from individual facts through reliable hypotheses. This road has been followed ever since by all the masters on the subject, down to the present day.[1]

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[1] *Works of Thomas Reid,* edited by Hamilton, foot-note p. 712.

3. Definition of Logic. Let us make clear the thing we are about to treat of. The object of Logic, speaking in general terms, is accuracy of thought in any of our mental operations, that is to say, accuracy in simple concepts, in judgments and in reasonings. This is what directly concerns us. However, to demand similar adjustments in the proper use of all cognitive faculties, inasmuch as they make for the attainment of truth, is not outside the field of Logic. The methods of induction, for instance, depend largely on precise observation of facts.

Now accurate or correct thinking implies that we must conform our mental processes to rules or standards of thought. To enquire, therefore, into those fundamental rules is one of our main concerns.

Having this outline of the object of Logic before our minds, we may lay down its definition as follows: Logic is a science which directs the operations of the mind by means of laws and principles that make for the attainment of truth and valid thought. Briefly: The science of correct thinking.

Logic is a science, for it makes up a systematic body of knowledge regarding the ways and rules of correct thinking and the estimation of evidence used in the various scientific methods.

Logic is a normative science, guiding our mental discourses by consistency to the goal of truth. In this peculiar character Logic is quite similar to Ethics, the science of directing the will by right conduct to happiness. It likewise resembles Aesthetics, which guides sentiment through precepts of good taste to rational pleasure.

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[1] For a brief history of our science, see *Elements of Logic* by Richard Whately, Introduction, pp. 2-7.

The Science of Logic is both theoretical and practical. In the first sense it lays down and criticises the general and particular laws of thought, and in the latter sense it comes down to practical analysis. In fact Logic is constantly being applied. For instance, it often occurs that in order to prove the truth of a certain proposition, we take the contradictory proposition and prove that to be false. In short it is Logic that analyses and determines both correct and incorrect inferences. According to the old terminology there is *Logica docens* and *Logica utens*.

Logic directly deals with thought. And what is a thought ? In its simplest form, it is an image or representation of a thing in the world around us. This mental representation, let it be noted carefully, is immaterial, and totally distinct from sense-perception—which is material. To make the difference manifest, look at an object, say a point or a line marked on the blackboard. This is what you perceive and qualify as white or red, thick or thin ; and this you picture also in your imagination. But the concept of a geometrical point or line is quite different. The teacher is careful to explain what they are. A point, he says, is a mere position in space without dimensions ; a line is an extension through space with one dimension only, length. Thinking here consists in realising a material object in an immaterial way by representing a point in general, or a line in general ; and what is here said of thought as a simple concept, is applicable to every process of thinking—for each bears the character of being immaterial and universal.

Other words in the definition require some explanation, namely, valid thought and logical truth. Validity of thought means consistency of thought with thought, either on account of implication, or because one depends upon the other. For instance, it is valid to say of some metals that they are conductors of heat, because all metals are; and all includes some. But it is invalid to hold that an event has happened, merely because it might have happened—for there is no inference from possibility to actuality.

Logical truth, however, means a great deal more than mere consistency between thought and thought, as we shall see later on.

It is curious to observe that logicians do not agree in defining Logic. The reason, however, must be traced to their different views on philosophical questions connected with our science. The reader may compare and discuss for himself the following definitions by authors of various schools:—

(1) "Logic is a science and art directive of the very act of reasoning, by which man in the exercise of his reason is enabled to proceed without error, confusion or unnecessary difficulty." [1]

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[1] "Si igitur ex hoc, quod ratio de actu manus ratiocinatur, adinventa est ars aedificatoria vel fabrilis, per quas homo faciliter et ordinate hujusmodi actus exercere potest; eadem ratione ars quaedam necessaria est, quae sit *directiva ipsius actus rationis, per quam scilicet homo in ipso actu rationis ordinate faciliter et sine errore procedat. Et haec ars est Logica, id est, rationalis Scientia,* quae non solum rationalis est ex hoc, quod est secundum rationem (quod est omnibus artibus commune); sed etiam ex hoc, quod est circa ipsum actum rationis sicut circa propriam materiam." St. Thomas, in his commentary on Aristotle's logical treatises, Leonine Edition, Vol. I., p. 138.

(2) "Logic is the art of directing reason aright, in obtaining the knowledge of things for the instruction both of ourselves and others."[1]

(3) "Logic, in the most extensive sense in which it has been thought advisable to employ the name, may be considered as the Science, and also the Art, of reasoning."[2]

(4) "Logic is the science of the operations of the understanding which are subservient to the estimation of evidence."[3]

(5) "Logic is the reflex-study of the order which needs to exist in our judgments, inferences, and more elaborate reasoning processes for them to lead us to truth."[4]

(6) "Logic is most comprehensively and least controversially defined as the analysis and criticism of thought."[5]

4. Logic a Science and an Art. Logic is both a Science and an Art; and it will be useful to place these two notions in a clear light.

Science in a proper sense (not restricted to natural or physical matters), is a system of doctrine built on principles and demonstration; in other words, theoretical knowledge.

Art, generally speaking, is the production of work guided by a code of rules. Whoever possesses art is an artist or a craftsman. The two are not easily found separately to-day—because nowadays almost every art is improved by scientific research. On the other hand, speculative knowledge tends to be practical and useful in production. The notions, however, of Art and Science are distinct, and refer to departments of knowledge having different characteristics :—

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[1] *The Port Royal Logic*, translated from the French by Thomas Spencer Baynes, 5th Ed., p. 25.

[2] *Elements of Logic*, by Richard Whately, p. i.

[3] John Stuart Mill, *System of Logic*, Vol. I, p. ii.

[4] Mercier, *A Manual of Modern Scholastic Philosophy*, Vol. II. *Logic*, p. 135.

[5] W. F. Johnson, *Logic*, Part I., p. xiii.

(*a*) Art is chiefly learned by practice, Science by study ; compare in this respect Geometry and Music.

(*b*) A Science is based on fixed and immutable laws ; Art is ready to change, and actually changes in the course of ages and among different people and nations.

(*c*) The object of Science is the reality of things which already exist ; Art is concerned with the creation of things.

These characteristics apply fully to Logic ; for it lays down the rules of correct thinking as something objective and unchangeable, and points out the reasons of these rules ; and then the same rules become a practical guide for the mind in the acquisition of truth. To use a comparison : As Medicine is a Science of the principles of sound health and an Art of healing, in like manner Logic is a Science of the principles of correct thinking, and an Art that imparts the habit of correct thinking and the avoidance of error.[1] In fact the Art of Logic as a practical application of the rules is far more useful than the Science of those principles. For centuries Logic was considered as synonymous with Dialectic, or the Art of argumentation, rather than as a rational science.

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[1] This point is skilfully developed by Richard F. Clarke, S.J., in his *Logic*, Ch. II.

A peculiar feature of Logic is its universality. It deals with all correct thinking ; and as all Sciences and Arts ought to be correct, it can be said that Logic is the Science of sciences and the Art of arts.

5. Division of Logic. In the operations of the understanding there is a formal and a material element. The matter of thought is the object we think about, while the form is the way in which the mind thinks of the object. The matter of thought may remain, whilst the form varies ; and vice versa, the same form of thought may be applied to various kinds of objects.

This distinction is very much the same as that made in Arithmetic between abstract and concrete numbers : the numbers 3, 4, for instance, and the operations with these numbers, say 3 plus 4 or 3 times 4, are forms, the matter of which may be books, apples or anything else.

In Logic the nouns All, Some, None, and the like, and such expressions as All S is P, No S is P, Either S or P, if S then P, etc., are simple forms of thought that exist only in the mind, and constitute the mechanism for working out the validity and consistency of our reasonings.

Logic is said to be material when it takes notice of the matter itself. Thus for instance in the Definitions and Predicables we look at the matter of our thoughts, and decide what is essential or accidental. Similarly in Induction we have to analyse concrete facts and their relations.

Formal Logic, then, studies the conditions of thought that make for consistency ; Material Logic considers the conditions of the things themselves.

Another very common distinction is that between

Deduction and Induction, which nearly coincides with the previous one. For deductive reasoning means that from a given statement we draw another which is implied in it. [Logical implications are usually expressed in symbols, for the sake of simplicity.]

Inductive reasoning, on the contrary, is material because of its starting point, which consists in gathering facts and trying to discover universality in them. For instance, by observing carefully that a given piece of gold is not attacked by acid, we are led to believe that no other piece of gold will be.

This division of Logic into two parts, one deductive the other inductive, can be soundly maintained ; but not as a hard and fast division. The two departments of Logic are not independent of each other. It would be wrong to say that Deduction is purely formal, and Induction purely material. In the former we look very often at the material contents, and are obliged to do so ; and in the latter we must rely on formal principles—of which much more will be said later on. The division merely indicates two main functions ; Deduction is analytic of the form of thought, whereas Induction is synthetic, and constructive of terms and propositions.

Another division, (the traditional one) is into Dialectic and Critic. Dialectic arranges our concepts for the purpose of argument and consistency ; Critic studies the process followed in the methods of the various sciences to arrive at truth. These two parts are distinct, not only in the matter and manner of treatment, but also in their historical development. Dialectic is to-day substantially the same as was laid down by Aristotle ; while Critic embraces all the modern methods of investigation initiated by Bacon in his

Novum Organum, and by Descartes in his *Discourse on Method*.

6. The Scope of Logic. It will now be well to state more explicitly the aim of Logic. It is not to increase the stock of our knowledge on particular subject-matters; this we receive from every branch of Art and Science. Nor is it to prepare a man for any particular discourse. Our aim is to train the mind in exactness of idea and correctness of reasoning. The field is boundless, extending as far as the range of human knowledge. We learn directly how we ought to think; and indirectly we try also to bring up to that standard the many ways in which people think wrongly.

This is the immediate end in view. Logic lays down the general laws of thought and the various kinds of rules for correct thinking; and every part of Logic points to that end. A further scope, and (so to speak) the fruit of Logic, is the attainment of truth. As no man is free from errors, mistakes and inconsistencies, a constant effort towards correct thinking, so as to create a habit of it, is the shortest cut to truth.

7. The Cognitive Faculties. Here one might ask: What are the activities which have to be directed? The question calls for a brief account of the various faculties or activities of the mind that are at work in the attainment of truth, or play a part in the complex process of acquiring knowledge. It appertains to Logic to give rules for them all; hence we need some acquaintance with them.

Observe, first of all, that we are in contact with the reality of things around us by the exterior senses of the body, located in their respective material organs. By means of these five faculties we are able to apprehend exterior objects by producing within ourselves a sensuous cognition of individual things. The internal phenomena are perceived by internal senses: that is to say, the imagination forms and transforms images of things which have entered through the windows of the exterior senses. The sensitive memory recalls past sensations stored up, making them ready for use. Instinct is the sense of what makes for well-being, acting as a sort of judgment to select or reject objects as may be convenient or otherwise. Sense-consciousness refers to ourselves all the various sensations received, and makes us aware of them and of their objects; to that end it occupies a centre to which the other senses are united as it were by so many radial lines. It is plain that all these faculties are material and organic, belonging to one and the same individual subject. By their functions a sort of reflection is reproduced within us representing the outer reality.

Notice the difference between perceptions and images of the imagination. The former are actually determined and caused by the objects themselves; but the latter are merely a product of the interior faculty that forms images and plays with them in a thousand combinations.

The sensitive faculties just described belong to us in common with the more advanced species of the animal kingdom; and they present to us single objects only. But knowledge is a higher perception than that of single objects and their qualities. Above the sensitive activities there is in man the most noble endowment of nature: intellect, or the power of understanding.

This inorganic faculty apprehends material objects in an immaterial way, and similarly conceives truths and qualities which do not fall under the domain of the senses ; such are the notions of virtue, law, duty ; the relation and attributes of things, the postulates of science; etc. [The process of intellectual cognition is variously explained by philosophers, according to their different systems of Psychology.]

Immaterial Consciousness and Memory are only different aspects of the same power of understanding ; and their functions are analogous to those of the sensitive consciousness and memory previously mentioned.

Two important points are to be observed. One is the harmony between the two groups of faculties, which is due to the fact that in man there is only one individual principle of life, the soul. The second is the constant dependence of the intellectual power upon the functions of the sensuous activities. Consequently a defective disposition in the organs must tell on the working of the mind ; and vice-versa, mental activity imposes a strain on the nervous system which results in the particular sensation of brain-fatigue.

The above sketch refers to the faculties of cognition by which objects are brought home to ourselves. But this is only one-half of the system of human activities. The other half consists of a group of appetitive faculties, by which we are drawn towards certain objects and repelled by certain others. Among these appetitive faculties the will holds a place above and totally distinct from the sensuous inclinations.

By a constant repetition of acts in the same direction the individual acquires a natural bent or disposition called habit, appetitive or cognitive. Both kinds may be good or bad ; but in any case, a habit merely implies facility in the performance of a given kind of action.

Note that the full treatment of mental faculties or activities, as to their order and physical nature, belongs to psychology, not to logic. The matter of habits pertains, under various aspects, to Psychology or Education.

8. Logical Truth. As already pointed out, the great purpose of Logic is truth; that is, to teach us how to attain the knowledge of truth. And what is truth? According to St. Augustine, and with him the traditional school, truth is what a thing is; that is to say, truth is reality itself.[1] We possess true knowledge, therefore, if we know things as they are. For example: That the earth revolves upon its own axis in the course of twenty-four hours is an objective truth. My judgment that the earth revolves and causes thereby the succession of day and night, is a formal truth. To hold that it is the sun that revolves around the earth, is an error. Wherefore, logical truth is nothing else but the conformity of our understanding to the reality of things, or to the objective truth.

Observe carefully the difference between consistency and truth. One's mind may be quite consistent with itself and yet not be true; for consistency means conformity of one thought with another thought, or agreement of one statement with another statement. Now this, strictly speaking, has no reference to reality. If it happens that things are otherwise than our reasoning takes them to be, then our train of reasoning is consistent and yet untrue.

This distinction is fundamental in Logic, as will be shown later. Meanwhile the reader is cautioned against the mistake of those who make true knowledge consist in the combination of ideas, assuming that ideas and not things are the immediate objects of thought.[1] This view confuses consistency with logical truth ; it is one thing to be formally correct, and another to be logical and true. Observe that the purpose of the understanding is to place our personal being in communication with reality by the knowledge of things. If things are not referred to, communication ceases, and knowledge is null and void.

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[1] "Verum est id quod est." *Solil.*, Lib. 2, Cap. V.

We have considered truth in two ways, as in things themselves and as in the mind. There is yet a third way, and this is called Moral truth, namely, conformity of speech with mind, in this sense, that we voluntarily speak out what we know. The opposite of objective or Metaphysical truth is absurdity or contradiction ; the opposite of Logical truth is error ; the opposite of Moral truth is a lie.

9. States of Mind in Relation to Truth. Our states of mind in relation to truth may be threefold : certitude, probability and doubt. Certitude means that we are firmly convinced of a proposition, because we have strong evidence for it. Probability means that we have a fair amount of evidence in favour of the proposition, but not enough to be altogether conclusive. Doubt means that we have no evidence worth speaking of either for or against the proposition, or else conflicting evidence which points both for and against it. Certitude means knowledge ; probability gives only opinion ;

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[1] John Locke avows this misleading view of knowledge. He says : " Since the mind, in all its thoughts and reasonings, hath no other immediate object but its own ideas, which it alone does or can contemplate, it is evident that our knowledge is only conversant about them." *And Essays concerning Human Understanding*, Book IV., Chap. I., p. 1.

doubt merely raises a question till further evidence leads either to probability or certitude.

It is from these relations of mind towards knowledge that we describe our statements as doubtful, probable or certain; because in reality they make them so. Certitude, however, well grounded on objective evidence, is the one sign or criterion of truth. [How these various states of mind are brought about is a matter for Psychology to explain.]

10. Logic and Language. Knowledge is expressed in oral and written language as a natural conveyance of thoughts to others and from others to us. Language, then, falls within the province of Logic inasmuch as it is an expression of our thinking.

It does not belong to Logic, but to Psychology, to analyse the ideological connection between thought and language; that is to say, the very important fact, that in thinking we speak (interiorly) and in speaking we think. The fact is that we need signs to fix and express ideas; and these signs, oral or written, are human language.

Observe, however, that language is complex, for it embraces something more than thought. Take the unit of language, a sentence. In it there is propriety and arrangement of words, which belongs to grammar; there is elegance and grace in saying things, and likewise force and emotion, all of which is Rhetoric; there is also thought, the direction of which pertains to Logic. The relation, then, of Logic to Language is not so simple as it looks at first sight, and needs some distinction.

Language, regarded merely as grammar, is practically nothing else but logical relations expressed in the parts of speech. This seems to be true of universal grammar, leaving aside minor details concerning particular grammars ; for instance, that the singular personal pronoun should be capital I, and that the adjective should precede the substantive, is mere English grammar. But the parts of speech, nouns, prepositions, verbs, etc., are no other than thoughts or logical relations embodied in words.[1] Rhetorical language, be it Poetry or Prose, is under the direct guidance of Logic only so far as accuracy of thought is concerned. The ornamentation added here and there to make the expression beautiful and charming to the hearers, belongs to the Art of speaking well. Logic and Rhetoric have two different ends in view : to carry conviction is the one, to persuade is the other. The means employed by the two Arts are also different, but both so blend together that one helps the other for the purpose of argumentative composition.[2]

This much to show what part of Language Logic considers as its own. Looking now at the current use or abuse of Language, we may notice particularly that it helps in many ways, but also often hinders the function of Logic.

Language helps to correct thinking because :

(*a*) It serves as a guiding thread in the labyrinth of ideas, which enables us to analyse complex reasoning. In short, Language facilitates reflection.

(*b*) It affords a common source for the formation of concepts ; i e., by using the names we get the ideas.

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[1] This point is further developed by W. F. Johnson, *Logic*, Part I., Introduction, p. xxi.

[2] For further information on this point see *Elements of Rhetoric*, by Richard Whately, Introduction, pp. 6-9.

(*c*) It shortens the process of thinking by giving ready-made definitions, and in general the connotations or attributes that make up an object.

(*d*) It is a direct means of communicating thought.

(*e*) It is a means of recording thought.

At the same time Language is a hindrance to thought because:

(*a*) Many words, on account of their being capable of different meanings, are misleading; for instance, court, law, church, foot

(*b*) The concept conveyed by a word changes with the progress of knowledge or with current use, e.g., prevent, conceit, worship.

(*c*) Terms often acquire an applied or metaphorical sense, e.g., to wield the sceptre, to tax one's strength.

(*d*) Many so-called Synonyms bear different shades of meaning which ordinary readers will not grasp.

This double aspect of Language will be referred to in the application of logical rules; not with a view of trying to change Language, but to help towards a correct use of it. Language will always remain an expression of the whole man with his imagination and feelings; subject therefore, to varying subjective dispositions, prejudices and false generalisations.[1]

11. How Logic Stands to other Sciences. Logic has no object-matter of its own, as Chemistry, Geography and in fact every other science has. It is characteristic of Logic to apply itself to every kind of knowledge. In this sense, then, it is equally related to all kinds of learning. Observe, however, that in another sense there seems to be a special Logic for every science. And why special? For the simple reason that each science has its own method. There are many kinds of truth, as there are many kinds of reality, and we study each of them in a different way. The process by which we learn Mathematics is not the same as that followed in Sociology or History. Consequently, the rules for Statistics and Probabilities are not those of the exact sciences. Logic, then, as applied to **Methodology,** admits of a variety of rules and precepts.

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[1] The Relation of Language to Logic is explained by J. Welton in his *Manual of Logic*, Vol. L., pp. 1-9.

For a fuller explanation see *An Outline of the Necessary Laws of Thought,* by William Thomson, Introduction, pp. 26-46.

Nevertheless Logic is one unchangeable science, binding together under the most fundamental laws the particular ones that make for correct thinking. A comparison will make this clearer. Each plant, for instance, bears appropriate leaves, flowers and fruit, and so does each of the big trees. The processes and results, though quite different, obey the laws of vegetable life that make the science of Biology.

In another respect, as a mental science, Logic is closely related to **Psychology**; in fact both cover partially the same field, namely, the operations of the understanding. The aspect of these operations covered by each of the two sciences is the following: Logic looks at the cognitional import of those operations, considering them as instruments of learning; that is to say, what we do or can do with them in the acquisition of knowledge. Psychology studies the same operations in their constitutional nature; just as Physics and Chemistry, for instance, study the constitution and properties of matter. Moreover the province of Psychology extends to all other activities of human life, besides those of the understanding.

Logic, again, as a philosophical science, has a close connection with **Metaphysics.** In every chapter of Logic we touch upon questions and presuppose principles that rest for further explanation on Metaphysics. For instance, the general Laws of Thought, the Laws of Nature, the origin and universality of ideas, are not criticised by Logic ; their ultimate reasons must be referred to metaphysical speculations. For the rest, there is a parallelism between the two sciences, in the sense that Metaphysics has for its object the fundamental constitution and order of things in themselves, while Logic attends to the same order of things as they are in the mind.

Logic, furthermore, as a rational science, stands in contact with **Epistemology** or the Theory of Knowledge ; and this is particularly realised in dealing with Induction, where the criticism of human testimony and the basis of Probability are dealt with. The two sciences, however, are kept separate ; for though they cover some common ground, Epistemology embraces the critical study of all the sources of cognition from the view point of the theory of knowledge.

We have reserved for the last place some remarks on the relation of Logic to the science of **Mathematics.** Let us call to mind that our science deals with forms of thought and their implications expressed by affirmation and negation, *is*, *is not* ; by conjunctions such as *if*, *either*, *and* ; by adverbs like *steadily*, *soon*, *never* ; by prepositions such as *to*, *from*, *in*, and so on. All these forms of thought are most general, being applicable to the subject-matter of any moral, social or physical science. Now it happens that some of these logical relations are capable of exact measurement in amount and in order of time and place by Mathematics, which is the science of quantity and order. On the basis of units of different kinds it has developed a wonderful system of formulas, or forms of thought; less general than those of Logic, but suitable to explain the material conditions of the world around us. In Arithmetic the units are mere numbers; Geometry is based on units of surface and volume developed from the idea of length; in Physics formulas are devised out of units of space and time, and so on. With all these elements innumerable calculations are made, which, being applied to practical cases, constitute the extensive branches of applied Mathematics.

It has recently been observed that Mathematics is an extension of Logic; that is to say, a science of forms of thought on a level of less abstraction than that of Logic. It seems however that Logic, being already universal in its application, admits of no extensions. It merely includes Mathematics among other sciences to which it can be applied.

Mathematics is concerned with numbers and quantities and directions in space, and enjoys the unique prerogative of always using exact, univocal terms both in subject and predicate; so that its propositions are more clear then those of other sciences. Hence instead of saying that Mathematics is at a lower level of abstraction than Logic, we ought to say that Mathematics is the most abstract science to which Logic can be applied. Mathematical statements, being abstract, are forms of thought that lend themselves to embody any concrete matter. The expressions : two and two equal four, is true for individuals of any kind. And if Mathematics are made use of to determine something concrete like the distance of a star, the figures themselves remain just as abstract as before. But, for all that, every mathematical statement is at the same time a statement of Logic, just as any statement on any other subject would be · or, in other words, what is form in Mathematics is matter in Logic. Hence there seems to be no need to imagine that there is any *peculiar* relation between Logic and Mathematics, different from that between Logic and any other science.

The difference, if any, lies in the fact that Mathematics as such, uses only Logic, while the other sciences require observation and experiment. Other sciences, again, being partially inexact, cannot always throw their knowledge into syllogistic reasoning, whereas Mathematics can always do so.[1]

12. The Utility of Logic. There is a natural and a scientific Logic. By the mere fact that man is endowed with the power of reasoning, he possesses also a natural disposition to exercise this power correctly in the acquisition of truth ; just as for man to walk upright is part of the faculty of locomotion. And experience shows that the greater the talent of a person for a special branch of knowledge, the more accurate and sharp is his reasoning.

To perfect this natural disposition by teaching the rules of correct thinking, their immutable foundations and the various ways of reasoning, is the purpose of our science. A noble task, if we look at the end in view, the possession of truth; most useful, if we consider the many errors into which most men fall—even those who have gone through a course of education; and necessary to unmask the fallacies of our adversaries. Note besides, that there is an art of sophistry, which calls for a counter-art of discovering and refuting fallacies, by showing what is right and what wrong in reasoning.[1]

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[1] On the Relation of Logic to Mathematics see Johnson, *Logic*, Part I., Introduction, p. xxiii.

Logic is not perhaps the first of the sciences in dignity, nor does it stand first in the order of nature; but it certainly occupies the first rank in disposing and preparing a man for the study of all scientific pursuits.

"If it were enquired," says Whately, "what is to be regarded as the most appropriate intellectual occupation of man as a man, what would be the answer? The statesman is engaged with political affairs; the soldier, with military; the mathematician, with the properties of numbers and magnitudes; the merchant, with commercial concerns, etc.; but in what are all and each of these employed?—employed, I mean, as men; for there are many modes of exercise of the faculties, mental as well as bodily, which are in great measure common to us with the larger animals. Evidently in reasoning. They are all occupied in deducing, well or ill, conclusions from premises; each concerning the subject of his own particular business. If therefore it be found that the process going on daily in each of so many different minds is in any respect the same; and if the principles on which it is conducted can be reduced to a regular system ; and if rules can be deduced from that system for the better conducing of the process,—then it can hardly be denied that such a system and such rules must be specially worthy of attention, not of the members of this or that profession merely, but of every one who is desirous of possessing a cultivated mind."[1]

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[1] Thus observes Cicero: "Nullam dicere maximarum rerum esse artem, cum minimarum sine arte nulla sit, hominum est parum considerate loquentium."

"Only unthinking people could say that there is no art for great things, when the smallest things are not without it." (Lib. 2, De offic.)

Logic, however, to be useful (as it claims to be), presupposes in the student, besides a liberal education, a fair acquaintance with the elements of the modern sciences. Without this knowledge, it will be difficult to understand the application and force of the rules of correct thinking—especially in the course of scientific induction, where it is necessary to follow the methods of each science in order to analyse their logical value.

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[1] See Whately's *Elements of Logic*, Preface, p. x.

CHAPTER II

TERMS

1. Three Operations of the Mind. Logic leads us to correct thinking. Now in the process of thinking we may distinguish three operations, namely, *Simple Apprehension, Judgment and Reasoning.* This does not mean that in each particular case we pass from simple apprehension to judgment and from judgment to reasoning ; but we should rather say that the intellect sees the concept, the connection of ideas, and the relation between judgments, sometimes separately, sometimes all at once. Each of these operations, however, must be considered separately and has to be analysed, if we want to keep order, attain facility and avoid error in the process of thinking.

2. Simple Apprehension. It is the mental representation of an object—meaning by object anything whatever, be it positive or negative, real or fictitious ; it is also called *concept, idea or notion.*

We obtain most of our concepts directly through the senses, and these are *intuitive ideas.* Other concepts are formed by analogy, and may be called *analogical ideas.* Examples of the first kind are the concepts of visible things, as water, room, tree, dog ; of the second kind, spirit, life, substance, virtue, law. We call the latter analogical or symbolical because, having no direct perception of them through the senses, we are bound to represent them after the manner of things directly perceived.

Such representations in the mind, whether *intuitive* or *analogical* ideas, may assume various degrees of perfection, as follows :

A concept may be clear or obscure.

A clear concept may be distinct or confused.

A clear and distinct concept may be adequate or inadequate.

A concept is clear when its object is plainly distinguished from all other objects, and **obscure** when it can be mistaken for some other somewhat similar object. Most intuitive concepts are clear ; but those of " democracy, planet, electricity," and generally speaking analogical concepts are obscure, so long as they are not explained.

A clear concept is distinct when I can definitely see its main parts, **confused** when they are somewhat blurred. For instance, the concepts of " triangle, room " are distinct, those of " capital, labour " are confused.

A clear distinct concept is adequate if it shows all that belongs to the object, **inadequate** if anything remains unperceived ; of simple things, say numbers, we possess adequate concepts.

Observe that the best concept is one that is clear, distinct and adequate as far as possible. We should strive to acquire such concepts of things by constant attention. Adequate concepts are few, and are the result of study and natural talent. How different, (to give an instance), is the conception of a " dynamo " among various pupils after listening to the explanation in a class of Physics ?

Having laid down the qualities of a concept, we proceed to its multiple divisions according to contents. But instead of dealing directly with concepts, we shall first treat of names for convenience' sake, namely, to correct at the same time thought and language.

3. **Names and Terms.** A **word** is a verbal expression carrying a meaning. There are as many kinds of words as there are parts of speech; but in logic we are particularly concerned with those which by themselves express a concept; viz., nouns, pronouns, adjectives and participles, and are called **categorematic** words. Other words (article, verb, adverb, preposition, conjunction and interjection) are called **syncategorematic**, because they cannot stand as terms by themselves.

Terms (from the Latin *termini*, ends), are the subject and predicate of a proposition. As all names may be terms of a proposition, names and terms are practically the same thing. Terms or names may be single worded, as table, island; or many-worded, as the House of Commons. Again, nouns and pronouns are usually subjects, while adjectives and participles are usually predicates in a proposition. **By a proposition we mean,** (for the time being), **a statement or declarative sentence,** for instance: "The weather is pleasant"; in which "pleasant" is the predicate and "weather" the subject.

Every name must fall under the following general divisions; that is to say, it must be **Singular** or **General, Positive** or **Negative, Concrete** or **Abstract, Absolute** or **Relative.**

4. **Singular and General Names. A Singular Name is** applied in the same sense to one object only, for instance: The earth, the present King of England.

General names are very often made singular by use of limiting words. Names of this kind are most significant, containing the attributes of the general name plus the notes of individuality. Thus, the general name ' boy " is changed into singular if I say, " The best boy in the class," and the meaning is larger in contents.

A General Name or a common name, applies in the same sense to each of an indefinite number of objects ; it is therefore a **class-name**. Notice that a general name may actually apply to very few objects and even to one, but is still applicable to many, for instance, Queen-Empress, Conqueror of England ; hence implication and applicability in the same sense are the characteristics of common terms. A general name is the verbal equivalent of a **universal concept.**

A Collective Name is given to a group of similar units, such as army, board of trustees, crowd ; and may be general or singular ; thus, the Vatican Library, the German Navy, are singular ; library, navy, are general.

Observe that a collective name may be used **collectively or distributively** ; in other words, we may say something of the collection as a whole, or of the units separately. Take for example :

The *citizens* raised a monument,
The *jury* finds the prisoner guilty,

where the subjects *citizens*, *jury*, are used collectively, because they are spoken of as one body. But in the following :

The *citizens* voted in the election,
The *jury* were kept without food,

the subjects are understood distributively, as if they were class-names, because the units are spoken of separately, that is to say; the *individual citizens*, the *jurymen*.

Again, the collective or distributive sense makes a different meaning in the following:

The Municipal Corporation was of this opinion,
The Municipal Corporation were of this opinion;

the former implies a collective decision, while the latter suggests agreement of many. In fact the second would be more logical, since opinion is an individual factor.

Substantial Names, or names of substances, as water, salt, gold, are general names, being applicable to different portions of the substance. When "water" is used to signify "all the water of the world," the name is singular, as in the following: "Water covers the greater part of the surface of the Earth."

Some logicians characterise substantial names differently:

Welton calls them a peculiar kind of collective names.[1]

Jevons holds them to be general collective names, being a whole of homogeneous parts of the same name and nature.[2]

Bain says that names of materials, earth, stone, salt, mercury, water are singular, since each denotes the entire collection of one species of material.[3]

There is, no doubt, some difficulty in assigning the logical characteristics to names of material substances. They may be taken as general names, according to the definition, but with a difference; they are not definite class-names like those given to natural or artificial kinds, say " table, flower." The individuals denoted by the names of materials are not fixed by nature. " Water " is divided into portions that can again be united, and any portion of it is called water.

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[1] J. Welton, *Manual*, Vol. I., p. 51.

[2] Jevons, *Studies in Deductive Logic*, p. 7.

[3] Bain, *Logic*, Deduction, p. 48.

5. **Positive and Negative Names** imply respectively the presence and absence of a quality, as " light, darkness ", " right, wrong." They are strictly opposed.

We must distinguish various degrees of opposition. Terms are said to be opposed when they cannot be applied together. The most important opposition is that between **contradictory** terms, namely those of which one positively denies what is said by the other ; for instance, " gain, not-gain " ; " silver, not-silver." Contradictory terms cannot simultaneously be affirmed or denied of the same subject. They exhaust the field of a definite line of thought. They constitute a complete division of all things, so that any object is under one or the other of the members of the division. It is not so between **contrary** terms, one of which denies more than is necessary to falsify the other, assuming the opposite quality, as in the following : " gain, loss " : " first, last " ; " high, low " ; " happy, unhappy." Such terms cannot be affirmed together, but may be denied together of the same subject. There is a middle term or even more than one middle term between them. Observe that the logical contradiction is made by **not.** The application of this particle to either of the two terms will be a test to discover real contraries or contradictories ; " dry " and " moist ", for instance, are contradictories, because " not-dry " is precisely " moist ", or " not-moist " is precisely " dry ". " Happy " and " unhappy " are contraries, since " unhappy " means more than " not happy ".

A Privative term implies the absence of a quality in a subject capable of it, as " deaf ", " dumb " ; and is contrary to the corresponding positive. Another opposition exists between *incompatible* terms, as " gold " and " silver " ; and similarly between two correlatives, as " teacher " and " pupil ". Other names are only partly opposed or *compatible* and therefore may be used together, as " employed " and " lazy ". If there be no difference whatever between two names, they are **identical,** as " George V." and " the Present King of England ".

The contradictory opposition is called by some logicians **formal,** while the other kinds of incompatibility are said to be **material.** Every term has its contradictory, but not necessarily its contrary.

6. Concrete and Abstract Names. A Concrete Name stands for a thing, or a subject of attributes. **An Abstract Name** is the name of an attribute considered by itself, and apart from the subject. Hence every abstract name has its corresponding concrete from which it was taken by mental abstraction. Thus, life is in living things ; kindness is in kind persons, but not every concrete name has the corresponding abstract ; it is a matter of language. There is a tendency, however, to coin abstract names out of substantives and adjectives alike, as in the following pairs : boy, boyhood ; visible, visibility ; triangle, triangularity.

It is somewhat difficult at times to distinguish the concrete or abstract use in a sentence. There are names, abstract in themselves, that admit of varieties. Such names as " activity, sensation, virtue, extension " may be used as concrete or abstract ; used as concrete they are general names, used as abstract they are singular.

Plurality or variety of kinds indicates that a name is used as concrete. In the sentences : " A foot is extension ; an hour is time ; temperance is a virtue ; anger is a passion." the predicates are used as concrete, meaning respectively, " a portion of extension, a measure of time, one of the virtues, one of the passions." Again, in the following : The education of this boy has been neglected, the name " education," being applied to the subject, is concrete.

Notice that all adjectives are concrete names, because they are considered as attached to things ; for when we say, " active, agreeable," we mean always " an active or an agreeable thing," etc.

7. **Absolute and Relative Names. An Absolute Name** implies no reference to any other name ; **Relative** implies reference to another and cannot be conceived without it. In fact relative names are referred to one another by a mutual relation ; hence they invariably go by pairs and are correlatives. Such names are found among adjectives and substantives ; some bear the same name, others different names. " Equal " is related to another equal, " friend " is related to another friend. " Employer " is related to " employee," " king " to " subject." The relation itself is an attribute, and we can always point out the fact which is the basis of the relation.

8. **Connotation and Denotation.** Every concrete general name implies a meaning that can be applied to many objects. Hence a name may be regarded under two aspects, namely, in connotation and denotation.

The Connotation or **Intension** of a name consists of those attributes or perfections that are included in the full meaning of the name.

Denotation or **Extension** of a name means all the individual objects to which the name is applicable. For instance, the intension of " a plane triangle " is space enclosed by three straight lines ; its extension includes all individual triangles past, present and future. Similarly, " house " implies the notion of a building suitable for habitation, and extends to all individual houses, because each is a building suitable for habitation. Connotation is the same as the nature of a thing or of a kind of things ; while the individuals of that kind or class of things are the denotation. The Connotation is essentially one, and exists in the mind as such ; but the Denotation exists outside the mind, and is not essentially one, except in strictly singular terms.

Proper names have no connotation, because they are applied to an object as a **mark of recognition**, or to point out an individual. **Abstract names** possess a minimum of connotation, namely, the *quality signified*. As to denotation, that of proper names is one ; that of abstract names is also one which coincides with the connotation.

9. **The Connotation of Proper Names** is a moot-point in Logic. The reasons for their non-connotation are (*a*) that proper names are given as a mark of recognition only, while a connotative name is applied to an object on account of attributes which the object possesses and the name implies. (*b*) Consequently a proper name may be given to any thing whatever ; but a connotative name stands only for a thing or kind of things in which the meaning is verified. This view is held by many logicians.[1]

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[1] See Keynes, *Formal Logic*, p. 41, and Welton, *Manual of Logic*, Vol. I., p. 53.

In favour of the connotation of proper names it may be argued that there is no difference between proper and significant names. Previous to their application both may stand for any thing ; and once applied to a definite object they raise in our mind the thought of that object. Thus the names " January, February," mean the first and second month of the year respectively, just as the names " my father, my mother " signify each of my parents. Such is the view of Jevons [1] and H. W. B. Joseph.[2]

Observe that if proper names are non-connotative, there will be names partly connotative and partly non-connotative, like the following : Mr. Brown ; Ex-president Wilson, which occupy a middle place between proper and fully significant names.

Bosanquet agrees with neither of those two opinions. He thinks that a proper name is significant inasmuch as it means directly an individual in its particularity, and secondarily its attributes. In other words, a proper name signifies an individual ; this is its meaning, and by this we come to know the attributes ; the proper name is a means to the end. Thus there is a difference between a proper and a singular significant name.[3]

W. E. Johnson declares that a proper name has a significance. In the proposition, " Waverley was the first novel written by Scott " we identify what is indicated by the subject and predicate terms respectively ; and what Waverley indicates is indistinguishable from what it means. Hence it seems to him legitimate to define a proper name as a name which means the same as

[1] Jevons, *Elementary Lessons in Logic*, pp. 42-43.

[2] Joseph, *An Introduction to Logic*, pp. 135-139.

[3] Bosanquet, *Logic*, Vol. I., p. 47.

what it indicates. In his terminology, a proper name is definable ostensively, but not intensively. In conclusion, while agreeing with the best logicians that the proper name is non-connotative, this, according to him, does not amount to saying that a proper name is non-significant, or has no meaning. It precisely means what would be indicated by a descriptive phrase like this : " the object to which I am pointing." [1]

10. The Connotation of Abstract and of Negative Names. Names used as abstract, according to some logicians, have both connotation and denotation. For an abstract name implies a meaning or content ; and this, no matter how simple in itself, is its connotation. On the other hand the abstract name refers to an attribute as conceived in the mind, which attribute constitutes its denotation. On reflection it appears that the connotation and denotation in this case are one and the same attribute—in other words they coincide. Hence abstract names are connotative and singular.[2]

J. Stuart Mill maintains, in accordance with his definition of connotation, that abstract names, as such, have no connotation, being the names of single attributes.[3]

A negative Name, say " not-white," has its meaning and its connotation, namely, the absence of the quality denied. As to its denotation, some maintain it to be indefinite or infinite, embracing the whole " universe " of things except white things. In this sense any object of thought, for instance, life, rain, feeling, language are included under " not-white." Other logicians prefer to limit the range of things not-white within the proximate genus of which white is a species, that is, coloured things. This, they say, is the " universe " understood and intended in speech. Similarly " not-horse " will imply any animal except a horse, and " no-thing " embraces objects of thought which are not in the universe of physical reality.[1]

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[1] Johnson, *Logic*, Part I., pp. 93-96.

[2] Joseph, *An Introduction to Logic*, p. 135.

[3] Mill, *System of Logic*, Vol. I., p. 31.

11. Kinds of Connotation. Terms may be used with a narrower or wider connotation. In order to avoid confusion, observe first that etymology is not connotation. Then let us distinguish between **logical. objective** and **subjective** Connotation. The **logical, or scientific** Connotation contains the most important attributes only, those on account of which the name is given, by which the object is recognised, and are common to all the individuals of the same class. Thus the logical connotation, being general, serves the purpose of science and definition. The **objective** Connotation or Comprehension embraces all the attributes to be found in an object, essential and accidental. The **subjective** Connotation is arbitrary ; it takes up from the object some attributes by which the object is personally known, or particular qualities by which we make it known to others. Such connotation is different in different minds according to their education and intelligence. Examples:

(*a*) *A circle* has its logical connotation as given in Geometry. Its comprehension is made up of all the properties of that figure together with the logical connotation. An ordinary mind will take it to be simply a round figure, as his subjective notion.

(*b*) *Gold* is scientifically a simple element as defined in Chemistry. It possesses besides many properties of its own.

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[1] Welton, *Manual of Logic*, Vol I., p. 68.

Anyone may distinguish gold by a particular property, say, its colour, or its insolubility by acid.

(*c*) *Education* has a rather complex connotation. Anyhow, its main characteristics will make the logical concept; all its aspects are included in a comprehensive concept; a particular aspect, say, mental education will be the subjective connotation.

From the above examples it is plain that the subjective connotation serves a purpose, when a thing is explained from one particular view-point.

12. Relation between Connotation and Denotation. The logical connotation of a term is decided by competent authority, and becomes fixed and current in the acquisition of knowledge, although subject to revision. The Connotation automatically determines the Denotation, namely all the individuals possessing the attributes of the connotation. Hence connotation and denotation are related, and it is usually said that they are inversely related, that is, if one increases the other decreases, but not according to any proportion. Consequently terms belonging to the same line of thought may be arranged in order of connotation or denotation. It may happen, however, that the connotation of a name increased or is decreased, the denotation remaining the same. The relation, therefore, may be correctly stated thus: "If the connotation of a term is arbitrarily enlarged or restricted, the denotation in an assigned universe of discourse will either remain unaltered or will change in the opposite direction." [1]

13. Universe of Discourse. This phrase is often made use of in Logic, and needs some explanation. Its first application occurs in the denotation of terms. As terms or names must have a meaning and this meaning refers to something, it is impossible to have a name empty ; that is to say, a name without a thing of which it is a name. The question then arises, where are those things we are speaking of ? The answer is that they are in the " universe of discourse " intended by the speaker. By " universe of discourse " is meant the range of objects to which the name is applied. This range of objects may be in different orders of reality, namely in the physical order, in fiction or mythology ; and the range may be limited or unlimited. Things there are in the sphere of thought, like species and genera ; other things are in dreams, in literature, in history, in the realm of facts. The objects referred to by the names : " griffins, ghosts, Olympian gods, fairies, characters of Shakespeare," are not in the physical order, but " student, tree, house " are names of things that we touch and see ; and names of non-entities, negations, contradictions, impossibilities express things that exist only in the sphere of thought.

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[1] Keynes, *Formal Logic*, p. 37.

Besides, within a given sphere, we may refer to the whole range or to part of it. Thus " Japanese houses, American trees, Students coming late " are names of a limited order.

So far we dealt with the " universe of discourse " regarding the denotation of names. The same notion helps us to understand the real meanings of propositions. Every sentence must be referred to a definite " universe of discourse." If I say : " No griffins exist," the statement is true for the universe of physical reality, and false for the universe of mythology or architecture. The expressions, " Everybody knows it, All say so " are true for the universe of people around the place.

The " universe of discourse " belongs to the matter of terms, propositions and arguments ; that is to say, whoever understands a proposition must take in the " universe " to which the term or proposition refers in a given context. But Logic requires that the " universe " should remain fixed during a whole argument, inference or explanation. Any change will cause confusion and inconsistency.

14. Ambiguity of Terms. *Ambiguity* is an uncertainty of meaning in words that bear more than one sense. The context usually determines the meaning in each particular case, but unfortunately not always. Terms, as to their meanings, are *univocal*, *equivocal* and *analogous*. There is no difficulty in *univocal* terms, as they always carry the same meaning ; such as " barometer " and other scientific names. *Equivocal* terms may stand for distinct notions ; for instance, church, foot, post, station ; the various meanings of names are given in good dictionaries. The greatest difficulty is found in *analogous* terms which can bear two meanings : one proper, and another that is a mixture of two on account of resemblance. In the following example : Sweet flower, sweet tune, sweet landscape, sweet poem, sweet friendship, sweet taste ; the term " sweet " conveys a proper sense in the last application, " sweet taste " ; but in the others carries a mixture of two sensations which in each case implies a different shade of pleasure resembling sweetness. Take another example : Healthy climate, healthy complexion, healthy exercise, healthy occupation, healthy literature, healthy body. Healthy " in climate " means producing health ; " in complexion," showing health ; " in exercise," developing health ; " in occupation," preserving health ; " in literature," developing health in the mind ; " in body," being healthy.

Other Ambiguities. The same univocal term may be construed in various ways. A plural or collective subject may be taken collectively or distributively. Examples :

(*a*) The people filled the hall (collectively).
The people stood up (distributively).

(*b*) The inter-arts students are more numerous than those of inter-science (collectively).
The inter-arts students write essays (distributively).

Again, a name may stand for a *real*, *material*, *grammatical*, or *logical* supposition. Examples :

Kings are rulers (real supposition).
Kings is a word of five letters (material supposition).
Kings is a plural name (grammatical supposition).
Kings is the subject of the proposition (logical supposition).

15. The Use of Terms. Why so many divisions of terms ? In order to ascertain the exact meaning of a subject or predicate in a proposition. What makes a term is not the sound but the meaning. The meaning in each case is only one, and this has to be singled out of other possible meanings in the name. Terms should be treated in Logic as univocal If the same name is employed in two, three or more distinct meanings, it should be regarded as two, three or more terms. This is precisely the reason why terms come into Logic, to make us aware of their proper meaning in building up a proposition—which meaning goes on in our inferences and through all the discourse.

This matter is easy enough, but the beginner needs some drill in a variety of examples, to be well acquainted with his first step of correct thinking.

CHAPTER III

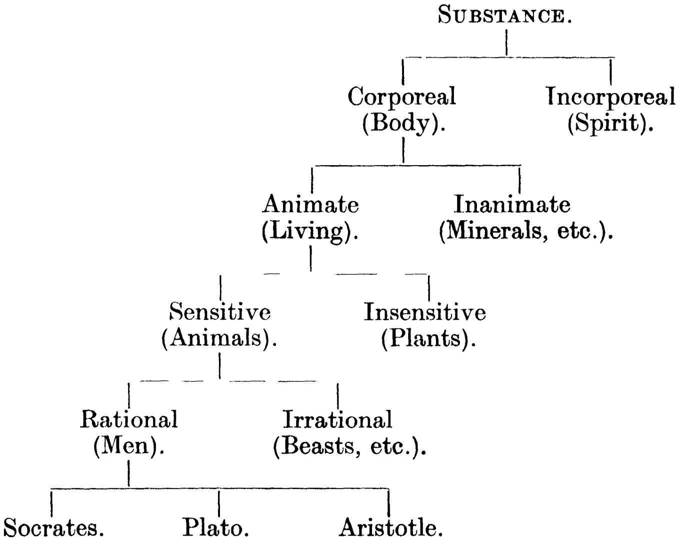
ANALYSIS OF TERMS

1. Mental Abstractions. Some account of mental abstractions, which are constantly used in the acquisition of knowledge, is necessary before entering into the analysis of terms.

Most things contain several qualities, all united in one individual. Thus, for instance, a piece of sugar is a substance that has a sweet taste, is white in colour, has a certain chemical composition and weight, is soluble in water, etc. Now the limitation of our understanding is such that we cannot grasp at one glance the comprehension of the name sugar. We have to look at the different aspects of the object one by one. Abstraction, therefore, is an operation of the mind by which certain qualities or attributes of an object are thought of separately from the rest. This looking at partial aspects of an object is common to all cognitive faculties. Our senses, for instance, each perceive only a portion of the reality : the eye only takes in colour and shape, the ear only perceives sounds, etc.

It is important to realise that whereas sense-perception is merely the perception of the single object, a mental concept is more than that ; for the mind by abstraction perceives objects under a universal form. The concept of man, for instance, is common to every individual, and I may say of each individual of that class, " he is a man." Furthermore, the intellect distinguishes in the notion " man " various degrees of perfection such as animal, living, substance. Going still further in abstraction the mind is capable of taking an attribute by itself, thus forming the abstract ideas of humanity, animality, rationality, life.

Here is a typical example of the work of abstraction, known as the *Tree of Porphyry* :


SUBSTANCE.
Corporeal (Body).
Incorporeal (Spirit).
Animate (Living).
Inanimate (Minerals, etc.).
Sensitive (Animals).
Insensitive (Plants).
Rational (Men).
Irrational (Beasts, etc.).
Socrates.
Plato.
Aristotle.

The above scheme is meant to exhibit a series of names : substance, body, living, animal, man, gradually increasing in connotation, while their denotation decreases. This subordination of names in which we pass from one to another by increasing the connotation is called a predicamental line. Observe the particular logical relations arising from this subordination. The name "substance," from which the line starts and beyond which we cannot go, is known in logical terminology as Supreme Genus, and the name "man" at the other end of the line, below which we cannot proceed, is called Lowest Species. The name "animal" is the Proximate Genus to the species "man," while the names "living" and "body" are both a Remote Genus in regard to the lowest species. Moreover, from the highest genus down to the lowest species each name is logically referred to the next as genus to species. A useful exercise for beginners is the arranging of various names in order of connotation. For instance, (*a*) house, building, structure, composite body, matter, or (*b*) plane triangle, plane figure, figure, extension, quantity. All concrete names may come under one or other of the various predicamental lines, according to the Aristotelian system of Categories of which we shall speak below. Names or things are said to be Homogeneous if, when compared to one another, they cannot be placed in subordination, because of their being undistinguishable by logical differences, or important attributes. For example, rain water, distilled water; rock salt, common salt; cast steel, wrought steel. Heterogeneous things, on the other hand, are so different that they cannot fit into the same line, for instance: book, feeling, sitting. A thing is said to be *sui generis* when it is so unique that nothing similar can be found in nature, for instance: the rings of Saturn, the solar corona, a peculiar style in architecture, a peculiar character of a man, etc.

Abstract generalising is not a deception, provided that it is grounded on the experience of a true similarity among the objects that constitute a class. Neither will it lead to deception, if our reasonings are confined to the limits of our abstraction. On the other hand, abstraction establishes generalisations which are a necessary condition for the advancement of knowledge. To think in universals is to think broadly ; to think in particulars is to think narrowly, though with deeper penetration.

2. **Predicables Based on Abstraction.** Mental abstraction lies at the root of predicables, or leads to the notion of predicables. If we conceive the attributes of a thing separately, those attributes, though separate and distinct in the mind, constitute the object what it is ; and we are right in affirming those attributes of a subject if present, or denying them if absent. Observe, however, that an attribute may belong to a subject in various ways ; as part of the connotation, or merely as concomitant to the connotation. If the predicate implies the whole connotation of the subject we have *a species* ; if it is part of the connotation common to other species, it is called *a genus* ; if it is part of the connotation determining the genus to a species it is *a difference* ; finally, if the predicate is merely concomitant to the connotation, but necessarily connected with it, it is *a property* ; if it is only concomitant and not necessarily following from the connotation it is *an accident.* Consequently there are five varieties of class-names that can be applied to a subject ; and these are called predicables.

The predicables are general names applied to certain predicates, not to indicate what they are according to their own meaning, but to express the relation they bear to the subject in an actual predication. Of the person Socrates I can say that he is a man, an animal, rational, mortal, sitting; and each of these predicates bears a different relation to the connotation of the subject.

The five predicables are described as follows:

(1) **A species** is a single class comprised within a larger group, as man is a single class of the larger group animal.

(2) **A genus** is a larger group that can be divided into smaller classes; as animal is a genus comprising man and the brutes.

(3) **A difference** is an attribute that must be added to the genus to complete the connotation of the species. In the same example, rational is the difference, because when added to animal it constitutes man. The three names, genus, difference, species, are correlative: if two are given, we can infer the third.

(4) **A property** is an attribute that belongs to every individual of a class, and is so deeply rooted in the connotation that it necessarily follows from it. For example, the fact of being moral follows from rationality in man, hence it is said to be specific property; mortality springs out of the genus animal, wherefore it is called a generic property in man or in the brutes.

(5) **An accident** is an attribute that accompanies the connotation but need not necessarily do so, and may be either separable or inseparable. An accident is called separable if in point of fact it is found in some individuals and not in others, or if (in case of some particular individual) it is observed to be at times present and at other times absent. An accident is called inseparable if in point of fact it is always found in every individual, or in a particular individual, even though it does not enter into the essential constitution of the

object, and does not follow necessarily from it. Examples : that a man is *ailing*, *running*, *talking*, that the Governor *is in Bombay* are separable accidents ; while hostility between cats and rats, blackness in crows (and generally natural colours of animals), *cloven-hoofs* in ruminants, *the date of birth* for each individual are inseparable accidents.

There is really a scientific line between property and inseparable accident, although it is often difficult to ascertain. The difficulty lies in our ignorance of the connotation of an object. In other words, we fail to see what connection the essential attributes bear to the non-essential. This connection is shown to exist in Geometry by demonstration ; in Physics or Chemistry it is found out only by experiment. An attribute of which we do not know the connection is called an accident. Thus " laughter " is a property of " man " because it follows from rationality ; the fact that " water dissolves sugar " is a property known from experiment ; but the peculiarity that " certain plants at evening close their leaves," we call an accident. Properties will always be co-extensive with the denotation of the subject ; inseparable accidents may fail to be so.

As a matter of exercise we may notice that the five predicables have their own fixed relation to a given subject ; but the same predicate may stand for one or other of the predicables where reference is made to different subjects. The name " animal " is a species in relation to " organic being," and a genus to " man." " Round " is a mere accident in a table or a window, and a difference in the round body called a ball. Again observe that of a singular significant name it is possible to predicate attributes that belong to any of the five predicables. For example, St. Xavier's College is a college (species)—an institution of learning (genus)—preparing for the University degrees (difference)—ruled by a principal (property)—founded in 1869 (inseparable accident)—gives M.A. lectures (separable accident). A genus as a subject may have properties and accidents as predicates. For example, virtue is praise-worthy, is rewarded. A species as a subject admits of a genus, a property, and accidents. For instance, temperance is a virtue, is healthy, is somewhat difficult. If a logician maintains that a proper name is altogether non-significant, whatever he predicates of it will be a separable accident.

One more remark may be useful to dispel difficulties. Modern logicians who profess extreme realism, make no distinction between predicables and classes of things, including among classes natural kinds and artificial kinds. So far so good. Observe, however, that a larger class and a smaller class need not be genus and species. The larger class will be a genus compared with the smaller class when the latter possesses a difference which is essential, or is part of the connotation. If the difference that causes one class to be smaller than another is an accident or a property, then the two classes will not be related as genus and species. Examples are familiar : body, large body ; rose bush, pink rose bush ; man, European man ; salt, sea-salt, are pairs of classes, one large, the other small, and not genus and species to one another. If we adhere to the Aristotelian notions of genus, species and difference, it is clear that species cannot be predicated of another species, so as to give two overlapping species. In the sentence : "Some poets are philosophers," subject and predicate are overlapping classes, not species. What is predicated here is an accident of individual poets. Observe finally that logical division, and the inverse process, classification, is made by predicables or classes, but not necessarily by genus and species.[1]

3. First and Second " Intentions." The old logicians distinguished appropriately between concepts of things and concepts of those concepts, that is to say, between what they called First and Second " Intentions," or what we now rather call " Notions."

First notions are **direct** ideas of the object, while second notions are **reflex** on the idea of the object, as if getting behind the idea and looking at it as an object inside the mind.

The five predicables are second notions formed by this kind of reflexion. The name " book," for instance, taken in first notion, means an object that we touch and see ; in second notion it refers to the previous concept as its object, and then it is declared to be a genus towards specific kinds of books, e.g., of philosophy, of history, of literature, etc. Again, in the statement : " The moon is moving round the earth," the predicate, " moving round the earth " taken in first notion, is a physical phenomenon ; in second notion, comparing that concept with the concept of the moon, the same predicate is a logical difference that distinguishes the moon from other heavenly bodies.

First notions are metaphysical concepts, second notions are logical ones ; and for this reason (namely, because they are a work of the mind) second notions cannot be predicated of individuals. I can say : This oak is a tree, but I cannot say: This oak is a species. The expression: The oak is a species of tree, is correct, because it means that the concept of oak is a species.

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[1] See J. S. Mill, *System of Logic*, Vol. I., Chap. VII. Of the nature of Classification and the five Predicables.

4. Definition and its Varieties. Definition is the means to analyse the contents of a thought, so that our notions, which are often indistinct or confused in our minds, may become clear and definite. The question may be raised as to what is it that we are supposed to define, whether the name, the thought, or the thing. Our view in this regard amounts to say that a name, a thought, and a thing are related to one another. The name stands for a special meaning which is our thought, and the thought refers to a real thing. Hence it seems immaterial whether we define one or the other. Strictly speaking to define means to set limits or boundaries for the sake of order and distinction, and this process properly applies to our knowledge. What we define, therefore, is our notions as they are or ought to be. A definition may be carried out in a variety of ways; but in all of them what falls under a definition seems to be the meaning of a name or the knowledge of a thing. An exception may be made of the nominal definition in which we analyse the etymology of a name rather than the meaning or contents.

The purpose of definition then is to make clear the meaning of a name, or what a thing is. Since our thoughts are naturally conveyed in language and invariably referred to the reality of things, there are several ways of making definitions. The meaning of a name is made clear by more simple words, or by inspecting the object itself, or by the analysis of the connotation into concepts which are implied in it, or finally, we have recourse to a description of the qualities which are proper to an object. We proceed to illustrate these various kinds of definition.[1]

(1) Ostensive definition consists in pointing out the thing itself; a simple and efficient manner of imparting to children the notions of things, that is, by object lessons. People in general acquire a good deal of clear ideas by inspecting the objects orderly arranged in a museum or in a picture gallery; and students entering a course of science obtain likewise clear notions of material objects by illustrations, pictures or specimens. The knowledge thus received is, no doubt, superficial, but none the less the method is suitable to recognise individual objects.

(2) Biverbal definition explains the meaning of a name by another name familiar in current speech; an easy method to be used whenever we meet with technical names, or those of foreign languages.

(3) Extensive definition tells us the signification of a concrete general name by examples. Thus we may say: alcoholic drink is beer, wine, brandy and the like. Psychical phenomenon is some such experience as taste, sound, anger, pain, pleasure.

(4) Logical definition endeavours to unfold in a concise manner the connotation of a name. It is a proposition in which the subject is the name to be defined and the predicate states the attributes that make up the connotation. Here the contents of the subject are analysed into distinct concepts, and the process is called by some logicians *analytic* definition. The nature of the process lies not so much in the analysis or enumeration of attributes, as in the logical concepts in which the definition is expressed. The high value of this

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[1] The special terminology has been adopted from Johnson's *Logic*.

traditional definition consists precisely in giving the whole connotation of a name by " proximate genus " and " difference." The proximate genus embodies the attributes which are common to all the species in the same line of connotation, while the difference adds that particular attribute which contracts the genus to one particular species. The statement, " a triangle is a three-sided figure " is a logical definition, because three-sidedness is the essential feature that distinguishes the triangle from all other geometrical figures. Other qualities of a triangle need not be stated, for these can be derived from the attributes given. Again : " man is a rational animal " is also a logical definition, for the word " rational " stands for the difference completing the species " man." The qualities of speech, laughter, growth and the rest follow from the definition. Observe that the essential elements " genus " and " difference " may be stated in many words. For instance : " capital is that part of a person's property (genus), which he employs for the sake of income (difference)."

The great merit of the logical definition is manifest, for it explains the connotation of the subject, and points out its relation to other subjects. On the other hand, it is difficult to lay down a good logical definition ; for to find out the precise genus and difference supposes a complete knowledge of the connotation of a name and a distinction of what is essential and accidental—a work accomplished only by long observation, comparison, analysis and generalisation or abstraction. As long as such work is not complete we must be satisfied with an imperfect definition subject to revision. The difficulty we speak of will be realised by attempting to define names like progress, civilisation, labour, etc., the connotation of which is controverted, complex or indefinite. Still more difficult, if not impossible, is the framing of a strict logical definition for kinds or species found in nature. Otherwise the logical definition is most expedient in deductive sciences.

(5) Descriptive Definition. Whenever an object is to be defined and its genus or difference are not available, its properties and accidents must be looked for. A descriptive definition explains what a thing is by means of qualities which, although exterior and partially common to other objects, yet when taken together are sufficient to mark out that particular object. Properties always present are obviously better than other accidental qualities for a description. For example : " Gold is a precious metal, yellowish and not dissolved by acids." There are other ways of describing an object. A statement of the manner in which an object is produced, is also a description. If I say . " A circumference is a plane figure produced by any taut cord revolving round a fixed point " ; or " An eclipse of the moon is effected by the earth intercepting the sunlight," I make Genetic descriptions. Similarly, a description may consist in the enumeration of the parts of an object, or in pointing out its causes or its effects. Descriptive definitions are very often the best that we can have not only in common matters but also in science.

(6) Nominal definition is properly concerned with the etymology of the name, that is, with its origin and formation, as when we show that " monogram " is a noun derived from the Greek language and composed of the words *monos* (alone) and *gramma* (letter). In this sense the nominal definition stands in contrast with the other

definitions, which are real, because all look at the meaning and more or less directly at the object which the name represents.

Sometimes mention is made of other definitions, namely, formal, real and scientific. The formal coincides with the logical because genus and difference are formal concepts. The real definition means that what is defined is a real concrete thing ; hence it applies to all kinds of definition except the nominal. A scientific definition is one made in terms and for the purpose of science ; it may be logical or descriptive, but in either case it must bear the marks of accuracy and precision. Moreover, one and the same object may assume different definitions in different sciences, according as it happens to be considered under its various aspects.

5. Rules of Definition. We proceed now to give the rules that will guide us in forming and testing definitions, and to illustrate the usual faults incident to this process of thought.

(1) A definition has to state the logical connotation of the name defined, neither more nor less. And by logical connotation we mean the most important attributes that constitute the nature of that thing for which the name stands. The errors incident to this rule may be by excess or by defect. A definition giving less than the connotation is said to be " too broad," and when the statement goes beyond the precise essential attributes is called " too narrow." Example : " A pen is an instrument for writing." The expression " instrument for writing " does not give the whole connotation of a pen, which implies writing with fluid ink. Hence the definition is too broad, as the same may be stated of instruments other than a pen. Again, " Alms is a gift

to the poor who beg for it." Here the statement is too narrow, for the precise meaning of the name "alms" does not necessarily imply the act of begging.

(2) The name defined is not to be included in the definition; when this rule is not observed there is said to be "a circle in defining," for instance, "dignity is the state of being dignified." The word "dignified" does not explain "dignity." It ought to say: the state of being exalted in rank.

(3) A definition must not be expressed in obscure, figurative or ambiguous language, otherwise we define what is unknown by something else not precisely known. The clearer the terms used in a definition are, the better. Example: "Proverbs are the wheat which remains after the whole world of talk and writing has sifted through innumerable minds." This symbolic and obscure statement becomes clearer, if we say that proverbs are practical truths ascertained by long experience. Again: "Kindness is the golden chain by which society is bound together." A poetical exaggeration which translated into plain language, amounts to this: Kindness is the quality of being gentle towards others.

(4) A definition should not be negative, if it can possibly be affirmative. The reason is because a statement of what a thing is not, does not tell us what a thing is. Thus: "Peace is the absence of war" is not a good definition; we should rather say, peace is tranquillity grounded on established order. It may happen, however, that the name to be defined has a negative connotation and then the best definition will needs be negative. For instance, "Darkness is the absence of light."

The rules just illustrated are generally directed to test the soundness of logical and descriptive definitions. But the first rule, at least, can be applied to any of them, for any reliable form of definition must endeavour to make clear and distinct the precise meaning of the name defined, or the attributes of the thing explained.

6. Limits of Definition. Every term which contains any meaning whatever is capable of some sort of definition ; at least by substituting some clearer word, or by a comparison with something else. But not every term is capable of strict or adequate *logical* definition. Thus :

(*a*) Proper names, having no connotation, are incapable of definition.

(*b*) Abstract names cannot be defined by genus and difference : for they connote no attribute, being the names of one single attribute.

(*c*) The supreme genera of things, which stand as categories, likewise admit of no definition, e.g., substance, quantity, quality, etc.

(*d*) Singular objects of experience are unique on account of their individual notes, which, neither together nor separately, can be a difference constituting a species.

(*e*) Terms of complex or indefinite connotation are not definable, because our knowledge of them is not clear enough to shape their connotation into genus and difference. For example : education, civilization, capital, labour.

(*f*) Names *sui generis* are undefinable by genus, so long as we find no similar objects in nature.

7. Division and its Forms. Another means commonly employed to analyse concepts is division. Definition explains the connotation, but division analyses the denotation. It is by division that we make out the species under a genus, or the varieties under a species. If divisions are carried out completely we come to know the various qualities possessed by the individuals of the same kind—whereby our knowledge is more thorough, approaching to perfection. The purpose, therefore, of division is to inquire into the varieties of individuals coming under the same class or connotation.

Logical division may be defined : The orderly distribution of the individuals included in a class or species. Accordingly, " man " may be divided into European, Asiatic, African, American, Oceanian ; or into Christian, Mahommedan, Buddhist, Brahmanic ; or again into Aryan, Semitic, Turanian. It is plain that in each case we take a different ground or basis of division (*fundamentum divisionis*) ; that is to say, each division is made according to one line of thought, or from one point of view : geographical distribution, religious belief, race. The various groups in each division are called members of the division (*membra divisionis*).

This view of division is a true one, and quite convenient for the purpose of a logical division. But we are also familiar with another way of looking at it. Speaking of division, one imagines a unit or an entire whole that has to be divided into its constituent parts. In this respect, which is also true, division is an enumeration of the parts that make up a whole or a unit. Now a unit may be of various kinds : a physical individual ; a moral unit, that is, an organisation ; a metaphysical unit ; or a logical unit, that is, a class of things to be split up into species or varieties. Of the logical division we have spoken above. Hence we have two kinds of division: (1) *Extensive,* in which the parts are merely logical parts, i.e., individuals classed under a species. (2) *Intensive,* in which the parts are real parts, i.e., individuals united into a corporation or parts united into a whole.

Physical division, or rather partition, is a simple enumeration of the various parts of which an object is naturally made up. For instance, a tree consists of roots, trunk and branches.

A moral division is the partition of a moral unit, say a family, an army, an association into the constituent members. For example, a family is naturally divided into parents and children; a government into executive, legislative and judicial departments.

A metaphysical division consists in regarding an object as an aggregate of qualities or attributes, which we separate in thought; as when in the object "man" we discriminate by abstraction the attributes of substance, corporeal, living, sensitive, animal and rational. The division of man would proceed as follows:

Man into animal and rational
Animal into body and sensitive
Body into extensive and substance.

Observe that metaphysical division coincides with logical definition, when the attributes mentally divided are essential; on the other hand, physical and moral divisions pertain to descriptive definitions.

Nominal division. Considering a name as a unit, we may divide it according to the various meanings that come under one name; as when I say that "humanity" means the nature of man in the abstract, and also the virtue of benevolence.

Other divisions. Divisions may be styled formal, real and scientific, just as we said before of definitions. The real is opposed to nominal; the formal coincides with the logical, specially if made by dichotomy [see below]; the scientific implies that it is made for the purposes of a particular science.

The relation between division and classification will be treated towards the end of Induction.

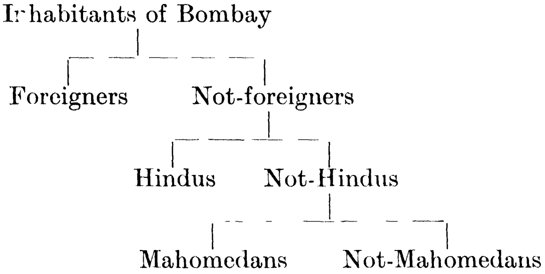
8. Rules of Division. The rules of a good division are usually stated as follows:

(*a*) The division must be adequate; that is, the members together must be equal to the whole which is divided—a rule that may be broken by excess or by defect. Thus, to divide the greater fine arts into architecture, sculpture, painting and music, is not an adequate statement of the denotation, which includes also poetry.

(*b*) The members of a division must exclude each other; otherwise there is useless repetition and consequent confusion. For example, the division of languages into classical, oriental and modern is faulty, according to this rule, because oriental languages may be classical or modern.

(*c*) The division must be orderly, so that the members at each step belong to one basis or line of thought; subdivisions, if any, must be placed in subordination, e.g., if I divide the inhabitants of Bombay into Europeans, Hindus, workmen and educated, I make a cross-division, putting in the same line members that should be distributed in subsequent subdivisions.

Division by dichotomy is exceptionally useful, as it avoids at once those defects pointed out in the rules. It consists of splitting up a species into two members contradictory to one another; and is consequently exclusive, adequate, on one basis and immediate. Examples:



Dichotomy is not necessary in obvious and simple matters. " But in less certain branches of knowledge," says Jevons, " our divisions can never be free from possible oversight unless they proceed by Dichotomy. . . . All the divisions of Naturalists are liable to this inconvenience. If we divide vertebrate animals into mammalia, birds, reptiles, and fish, it may any time happen that a new form is discovered which belongs to none of these, and therefore upsets the division." [1]

9. The Use of Definition and Division. We may declare at once that definition is the general and necessary means to obtain the primary knowledge of the things around us, which is naturally presupposed in any process of correct thinking. It is indeed commonplace to advise here that anyone about to speak on any matter whatever, ought first to make clear in his mind the precise definition of the particular subject he undertakes to develop. A concise statement of the view, or the aspect under which he considers the subject in question, will ensure from the start a mutual understanding between the speaker and the audience. Experience and reason alike go to show that any confusion left in the meaning of a term is bound to obscure the whole discourse. One may speak, for instance, of rights and obligations, of education and instruction, of things being possible and impossible, and so on ; but unless the meaning of those names be fully and distinctly grasped, the reasoning must needs go amiss. It is a matter of common observation that many discussions never come to an end, and fail to bring about the desired conviction. Now, the root of the evil lies, nearly always, in the fact that some fundamental notions are misunderstood or ill defined. There is no need of stressing the usefulness of sound definition, for this is evident from the constant recourse we make to dictionaries in order to find out the precise meaning of words.

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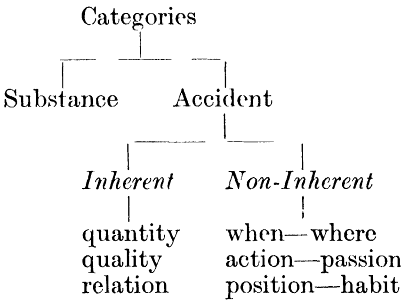
[1] Jevons, *Elementary Lessons in Logic*, p. 107.

Another remark to our purpose is that to memorise a definition is not enough. Beginners especially should make it a point to understand thoroughly the words used in a definition, and why these precisely and no others are employed. The reason is obvious. If a definition unfolds the connotation of a name or, in other words, states distinctly the essential attributes that constitute the nature of a thing, then we are in possession of the properties that follow from the connotation ; and very often these properties can be demonstrated from the definition itself. Besides, a good definition affords a key to the solution of difficulties, and a guide to discover relations and anologies leading to further conclusions. We should remember that definitions are not only the beginning, but also the end of knowledge, that is to say, the outcome of work carried on by generations trying to know better the nature of things. As human knowledge is ever growing, definitions are the best deposits we can have for the time being, but are subject to change and do actually change and become more perfect with the progress of science. The latest book on Physics, Chemistry, Geography or any other science contains definitions more correct or more complete than those in current use some years ago.

What has been said of definition is proportionally true of division, which completes the knowledge of a thing in regard to its accidental qualities and varieties found existing among the individuals going to make up the denotation of a name. Thus, in order to discourse properly, for instance, on the motor-car, besides the definition, one must learn by division the great variety of these vehicles as to their make, sitting accommodation, model, motor-power, etc. Similarly in the subject-matter of schools, after the definition, there still remains the study of the many kinds of schools under their different aspects, such as private or official, elementary or secondary, English or vernacular, etc. Any deficiency regarding division will render our exposition of the subject-matter inaccurate and incomplete.

Finally, a good exercise for beginners is to attempt and test definitions and divisions of general names, and to examine by the logical rules, those given here and there by hasty writers.

10. Categories. There is yet another way of analysing names or rather things, namely from the point of view of their nature. Aristotle was the originator of this general division of things, as they are in themselves, into prime heads which he called categories, and Boethius called predicaments. The ground for this division is their manner of existence. Thus, any real thing either exists by itself and not in another, in which case it is a substance ; or else it exists in a subject, and then it is an accident. Hence substance and accident constitute two supreme genera. Now, entities existing in a subject may be inherent, such as quantity, etc., or may be modes of existence referring to time, place, etc All the categories, therefore, taken together are ten, as follows :



Observe that in the mind of Aristotle categories are positive things and not negations. Moreover, they are most general names, or kinds of things, and to one or other of them everything must be referred. The same thing, therefore, may be a category, a predicate and a predicable, if examined from different points of view. Again, accidents as predicables are different from accidents as categories. An accident as predicable means an attribute that falls outside the connotation of the subject—as merely concomitant, be it a substance or not ; while accident as a category is physically so, on account of its way of existing in contrast to substance.

In other words, substance and accident as categories are contradictory terms, while accident as a predicable bears no opposition to substance; for instance: "India is a large part of the British Empire." Here the predicate as a category, or in itself, is a substance; but as a predicable it is an accident, for it is accidental to India to be a portion of that particular empire.

11. Systems of Categories. The Aristotle's categories just mentioned are concepts of real things divided on the common basis of actual being, so that each category represents a particular mode of existing. Besides, there seems to be a similarity between those categories and the parts of speech: Substantives (substance); Adjectives (quantity, quality); comparative names (relations); Active, Passive, Transitive and Intransitive Verbs (action, passion, position and habit); Adverbs (when, where). It is manifest that Aristotle's categories are deeply-rooted in speech, wherein probably they became embodied by long use in the ancient Grecian academies.

Nevertheless those traditional categories have met with opposition and controversy in modern times. It is no wonder. Categories are used in Logic as a groundwork for a clearer and more distinct conception of all things around us, but their discussion belongs to Metaphysics. Hence it is but natural that a new system of Philosophy will devise a new system of categories. We shall reproduce here some of them as a matter of illustration:

J. S. Mill arrived at the following result as a classification of all nameable things:

1st. Feelings, or states of consciousness.

2nd. The minds which experience those feelings.

3rd. The bodies, or external objects, which excite certain of those feelings, together with the powers or properties whereby they excite them.

4th. The successions and co-existences, the likenesses and unlikenesses, between feelings or states of consciousness.[1]

The reader will note that this classification is on the basis of phenomenalism; but as a logical distribution of all our notions of things, it may be discarded as irrelevant.

Kant's categories are altogether from a different point of view. They are not a classification of things, but of elements of thought native and necessary to the mind, and from which our various judgments spring. We discover quantity, quality, relation, and modality in our ways of judgment; hence he deduced the following scheme of categories:

Of Quantity—Unity, Plurality, Totality.
Of Quality—Reality, Negation, Limitation.
Of Relation—Independence, Dependence, Inter-dependence.
Of Modality—Existence, Possibility, Necessity.

Waiving the question of Kant's orderly distribution of the pure forms of the mind, we simply remark that it is right to study and distinguish the forms of judgment; the error lies in concluding that those forms cannot represent things in themselves, but only concepts or objects constructed by us.[2]

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[1] Mill, *System of Logic*, Chap. III., p. 83.

[2] See *Immanuel Kant's Critique of Pure Reason*. Translated by F. Max Muller, pp. 58-69.

Other philosophers, Descartes, Leibniz, Hume, etc., have devised different categories according to their various systems. As to them all I agree personally with the view of J. Devey : " Of the list of categories already given to the world the peripatetic is the only one which aims at taking a distinct place in logical science. . . . Aristotle's attempt, to say the least of it, is the most perfect that has been made." [1]

12. Schools of Logicians. The reader must have noticed that we have been speaking of thoughts, names and things apparently with some confusion. Now we must clearly distinguish them, and the views taken of them by different schools of Logicians.

Let us call to mind that the purpose of Logic is to direct the operations of the mind in correct thinking. The question therefore arises, what subject-matters are we treating of. To answer this question there have arisen three systems well known in the history of Logic ; Conceptualism, Nominalism, Realism. The controversy involves a difficult problem, namely, how to explain the universality of names and of predication.

The Conceptualists, of which Hamilton is a modern representative, hold that logic treats only of thought, naturally communicated to others by the use of names and propositions as symbols of thought. [To this the realists reply, that our thoughts refer to things in actual predication ; hence we speak of things.]

The Nominalists maintain that logic deals with language in the different stages of names, propositions and arguments. Whately writes : " Logic is entirely conversant about language. If any process of reasoning can take place in the mind without any employment of language, orally or mentally (a metaphysical question which I shall not here discuss), such a process does not come within the province of the science here treated of." A little later on he says : " In other words, rules may be framed that will enable us to decide what is, or is not really a term, really a proposition, or really an argument ; and to do this is to guard completely against the defect of inconclusiveness, since nothing that is inconclusive is really an argument." [1] [To this the conceptualists reply, that a term is its meaning, which is nothing else but the idea put in words. The rules, therefore, must needs be referred to thoughts, rather than to language.]

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[1] J. Devey, *Logic*, p. 349.

The Realists, among whom Mill, Bain and Venn are leaders, assert that Logic is concerned with relations of facts, physical, social or moral. A proposition, they say, is an expression of facts, not of mental concepts ; the statement, " grass is green," means that the thing, " grass," possesses the attribute " green." [To this the nominalists and conceptualists reply that such system is unable to explain the universality of terms and propositions.]

The solution seems to lie in combining the three systems into one, by saying that a term is a verbal expression of an object as conceived in the mind. Hence we deal primarily with thought, which represents things, and is expressed in language. Practically we deal with terms and propositions, but are constantly looking at their meaning as represented in thought ; and similarly we view our thoughts as representations of things. In other words, the test of thoughts and propositions, as far as correct thinking is concerned, is to be sought in the mind ; and the touch-stone of the mind is the reality of things. Hence for the purpose of truth, reality must be looked to. In conclusion, the rules of logic apply to thought, to language as representing thought, and to things as conceived in the mind. The three systems are essentially related. To be more precise, however, we may add that a general name may be looked at as a name, as a concept or as a thing. Grammar is primarily concerned with the name, Logic with the concept, and Metaphysics with the thing—three aspects of the name related to one another.

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[1] Whately, *Elements of Logic*, p. 37.

As to the element of universality (on which there is no end of dispute), the clearest explanation is to be found in Moderate Realism as defended by the Schoolmen. In this traditional school, the object of our mental notions is nothing else but reality as perceived. A universal concept (of which the general name is the verbal representation) is composed of two elements ; the object referred to, and the form of the mind. The form is not predicated of things ; it is only a means of predication. We predicate what we mean to say, the object itself. The universality, then, is a work of the mind, and from the mind is translated to language.

CHAPTER IV

THE PROPOSITION

1. The Second Operation of the Mind. In the acquisition of knowledge the human mind naturally proceeds from simple apprehension to judgment. Our first perception of a thing, as a rule, is under a general aspect as man, house. But by paying close attention to the object in its different aspects, we find that it contains certain properties and attributes, and does *not* contain certain others. As a natural consequence the mind when thinking of the object, *affirms* the properties, etc., found in it, and *denies* the other properties, etc., not found in it. This act, whether of affirmation or denial, is called a *judgment* deciding what a thing is or what it is not.

2. What a Judgment is. The advance from apprehension to judgment may be stated as follows : Apprehension *sees* what the thing is ; judgment *asserts* what it is (as seen). It is in judgment that I make the thing apprehended my own. I nail down the apprehension, so to speak, thus committing myself to the truth or falsehood of the assertion. A judgment, therefore, may be thus defined : A mental enunciation of what a thing is or is not. Observe (1) that this mental enunciation *presupposes* a perception of an object (simple apprehension) and consists in (2) a mental *assertion* about the object. Hence in every judgment there must be two simple apprehensions: (1) the object spoken of, and (2) some feature or characteristic or aspect or part of the object, which is attended to and affirmed or denied in the very act of assertion. Such is the way the human mind naturally proceeds in the acquisition of knowledge, namely by *composition* or by *division* of two concepts; and that is judgment.[1]

This decision of the mind, to be ranked as a *formal* or deliberate judgment, must follow as a result of some comparison in which it was ascertained that the object was what it was said to be; in other words, that the predicate belonged to the subject. Upon the clear perception of agreement or disagreement of the two concepts, the understanding assents and pronounces the statement. For instance: If I affirm that "a circle is round" on a previous perception that all the points of the perimeter are equidistant from the centre, my judgment is formal because I have fully apprehended the meaning of subject, of predicate and of their mutual relations. Again, my stating that "the moon is not self-luminous" on the evidence of its changes according to its positions towards the sun, is likewise a formal judgment.

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[1] The explanation of the act of judgment by composition and by division of concepts is a traditional doctrine. Thus St. Thomas says: "Intellectus humanus non statim in prima apprehensione capit perfectam rei cognitionem; sed primo apprehendit aliquid de ipsa, puta quidditatem ipsius rei, quæ est primum et proprium objectum intellectus; et deinde intelligit proprietates et accidentia et habitudines circumstantes rei essentiam. *Et secundum* hoc *necesse habet, unum apprehensum alii componere et dividere,* et ex hac compositione et divisione ad aliam procedere; quod est ratiocinari." *Summ. Theol.* I., q. 85, a. 5.

Judgments of that kind are called perfect, logical, predicative and subsumptive, because, as a rule, a universal predicate is affixed after analysis and reflection. Many of our judgments in daily life fall below that perfection, for the truth about the object is perceived almost without reflection,—as when I say: "It rains; He is my friend; This is larger than that; This measure is equal to the other," etc. These judgments, though most simple, because they touch an elemental fact, are essentially an advance on simple apprehension. The saying, "It rains" is a shortened form of "Rain now falls around me."

The judgment, properly speaking, is the unit of thought, or the minimum of real knowledge,—just as the cell is considered in Biology to be the smallest living organism complete in itself. Previous to judgment there is no statement of reality; but in judgment a pronouncement is made, which must be true or false. In fact the judgment is the most important of the three operations of the mind, because in it there is a statement of truth. The simple apprehension leads to a judgment, and the act of inference also leads to a judgment.

Many subtleties are involved in the act of judgment. For instance, how the mind bridges over between the real world and the ideal knowledge, is a problem much discussed by leading thinkers. The above notions, however, will suffice to guide the reader in the matter of propositions. The most important point for us is how to judge correctly. For that purpose the logical doctrine on judgment is usually applied to propositions, and rightly so; for language, as representative of thought, is (so to speak) official currency; and by checking its correctness we check thought as well.

3. What a Proposition is. As terms are external signs of ideas, so the proposition is a verbal expression of a judgment. It is called a proposition from the words *pro* and *pono* : I lay before you a judgment. A proposition is also called enunciation and interpretation, because by a statement the simple notion in the mind is evolved and interpreted into a perfect meaning. Thus, for instance, the simple notion " trade " is evolved into a perfect meaning by saying : " Trade is the interchange of goods." The proposition is equivalent to what is called in Grammar a *declaratory* sentence—the usual form of speech to declare that something is or is not. For instance : Gold is heavy ; Feathers are not heavy. Other sentences, the imperative and interrogative, are not propositions. If I say : " Clear the way ", I make no statement, but simply " command you to clear the way." The interrogative sentence consists in asking a question. Thus to say : " Has the boat arrived ? " is a short way of saying : " I ask " or " I wish to know whether the boat has arrived." In either case there is no statement of truth or falsehood, and therefore no proposition.

A proposition is the verbal equivalent of a judgment, and is correspondingly composed of two parts called the subject-term and the predicate-term. The *subject-term* is the name of the thing spoken about. The *predicate-term* is what is said about the subject. Comparing these two parts of a proposition with the parts of speech as recognised in Grammar, we notice that substantives or nouns, bearing by their nature the representation of things, serve as subjects, while adjectives and verbs, being determinations or attributes of things, serve as predicates.

The predicates can take a variety of forms :

(1) To state existence or reality of the subject. e.g. : I *am*=I *exist.* Fairies *are not*, for they *do not exist.*
(2) To state what a thing is as a whole, e.g. : This is a house.
(3) To state qualities or attributes, e.g. :
    The rose is red ; The sea is not calm,
    The fire glows ; No man endures for ever,
    The battle of Waterloo is a historical fact,
    What you say is not true.
(4) To state actions or active relations, e.g. : Milton wrote *Paradise Lost* ; No man creates.

Observe that a predicate always contains a verb ; the verb is essential in affirming or denying ; even to state a simple attribute of a subject we need the verb "to be," without which there would be no predication or no proposition.

4. The Logical Copula and the Verb. We have seen the two essential parts of every proposition, namely subject and predicate. Is there any *logical copula* ? We pointed out in explaining the judgment, that the human mind is naturally slow in the acquisition of knowledge. It states gradually to itself what a thing is, or is not, by so many predications ; in other words, it learns by composition and division. Now this mental act of composition and division is made explicit in the proposition by the so-called *logical copula.* There is no such thing in the mind—the judgment is a simple act ; nor in the things themselves—individuals are what they are ; but there is such a thing in the logical proposition, as a mere grammatical necessity. Logicians maintain that a proposition in strict logical form, (that is to say, fully analysed), ought to express the function of composition and division that takes place in the mind.

This function of the copula is expressed by the present tense of the verb "to be": *am, am not*; *is, is not*; *are, are not*. The verb "to be" may in ordinary language affirm actual existence of the object, as in the expression "God is." But when used as a copula it merely asserts the agreement or union or identity between the subject thought of and the predicate which is joined to the subject. Or if the copula is negative, it merely denies such agreement, etc.

In case of sentences made up with active verbs, e.g., "The horse runs," the predicate "runs" needs no copula in ordinary speech; but for logical analysis the copula has to be introduced thus "The horse is a running animal."

5. Time "in" and Time "of" Predication. Another function of the logical copula is that of expressing the present moment, or what the mind says *now*. Observe the peculiarity of a verb, viz., that it includes the note of time. By saying: Philip was the father of Alexander the Great, I predicate of "Philip" something of the past. But my judgment or proposition is laid down now. Hence in every proposition that is true, there is a time *in* and a time *of*.

Time *in* is the time referred to by the proposition for which the proposition is true; this belongs to the predicate, and is quite distinct in any tense other than the present.[1]

Time *of* is the time at which the proposition is uttered. This changes with the speaker, while the former is fixed with the proposition. For example: The tea is hot, He is not at home. Let both propositions be true at this moment, say, four o'clock; this time is the time *in* predication, for which the above propositions will always be true. One hour later those expressions are totally different propositions on account of the time *in* being changed; they are new propositions and may be false, while the former remain true for ever. For it will always be true that the tea *was* hot at four o'clock; a truth which at that time was, "the tea *is* hot." The time *of* predication, on the other hand, has nothing to do with the truth of the proposition, for it does not belong to the contents of the predicate.

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[1] See Bosanquet, *Logic*, Vol. I., p. 203.

Summing up, these are the reasons for introducing the copula in order to render a proposition in strict logical form: (1) to make explicit that a thing may or may not be predicated of another as the mind understands it; (2) to distinguish the present act of the speaker from the contents predicated; (3) to compare two judgments at the same time. Thus the copula is a means to analyse a proposition as to its logical truth or falsehood in a fixed universe of discourse. Examples:

(*a*) "I have fought a good fight." The predicate is freed from the present act of judgment by the logical form: "I (am) one who fought a good fight."

(*b*) "A brave soldier will not desert his post." The predicate is explicit in the logical form: "A brave soldier (is) a man who will not desert his post."

(*c*) "The river flooded the field." In logical form: "The river (is) a current that flooded the field."

(*d*) "He is healthy person; The battle of Waterloo is a historical fact" are expressions in logical form, for the copula and the predication coincide with the word "is." This, however, brings about the confusion of the time *in* and the time *of*, where unfortunately the word "is" combines together three distinct points: the present of the speaker, the present of reality and the copula of predication.

6. Matter and Form of a Proposition. We are now in a position to distinguish between the matter and the form of a proposition. **The form** consists in the copula, that is to say, the connection between the subject (the thing interpreted) and the predicate (the thing interpreting), and is expressed by symbols thus:

S is **P**, **S** is not **P**.

The matter is the reality referred to in the proposition, or embodied by the form. This form is the same while the matter changes. It is because of this form (which represents one of the general ways in which the mind apprehends reality), that we are able to construct that part of the science of Logic dealing with the implications of the categorical proposition. Other forms of propositions, that represent aspects of reality other than the categorical, will be studied later on.[1]

7. The Drift of a Logical Proposition. The logical proposition is made up of subject, predicate and copula. Which of the three is most important? The predicate. The understanding pronounces a judgment inasmuch as it apprehends the predicate. In fact we need not know the subject; it becomes known by the predicate, which gives information about the subject. Subject and predicate are two in mind, but in reality they are the same thing. In other words, the proposition is a product that coincides with the thing itself, since the mind unites things that are one in reality.

The subject of a proposition usually stands for a concrete fact apprehended through the senses, while the predicate is an attribute or adjectival notion applied to that subject, so that, (in the natural order of thought) the subject is richer in connotation and narrower in extension than the predicate, as when I say, " Water is liquid, air is fluid." The same rule holds if the subject is an abstract notion, for instance : Education is beneficial to society ; Prudence is a virtue. I may also say, " Some liquid is water ; A virtue may be prudence " ; but here the subject stands for individual instances of liquids or virtues. If subject and predicate stand for individuals, or for notions that bear no subordination to one another, there is no distinction between subject and predicate. I may equally say, for instance : " Some doctors are politicians " or " Some politicians are doctors." " Sunday is the first day of the week " or " The first day of the week is Sunday."

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[1] See Joseph, *An Introduction to Logic*, pp. 143-53.

The beginner may picture the subject as a box full of contents, (that is, connotation), which becomes known by successive predications or pronouncements. The basis for all propositions is the Law of thought called the Principle of identity. This principle says : Everything is what it is ; in other words, everything is identified with all its attributes and perfections. The mind, therefore, is right in predicating of an object each one of its perfections separately apprehended, and this is done by propositions. In a series of propositions like this : The earth is a planet, is spherical, moves round the sun, is inhabited, etc., I evolve by separate conceptions what the earth is, each predicate being the same subject differently conceived.

From the above exposition of the act of judgment, and of its representative the logical proposition, it follows that the logical copula is not properly represented by a sign of equality. The expression $A=B$ is primarily a relation of extension, not of intension ; it does not say directly that the attribute of B is included in the thing A or necessitated by the thing A, but merely asserts coincidence in quantity. The logical copula " is " goes further ; for it means identity of being between subject and predicate. For this reason the predicate is expressed in the concrete ; that is, in some sense involving the idea of the subject in it. We say : " The paper is white," and not " The paper is whiteness." [1]

One more remark. Writers who profess to be subjectivists explain the inclusion or subsumption of the subject in the predicate in the sense that one concept is included in another. If this inclusion is meant merely as subjective the view is wrong. Let us remember that propositions are an expression of objective reality ; and reality, not a form of the mind, is the object of human speech. By saying, The lion is fierce, John is my brother, Light is a wave-motion, I mean that things are so in reality. Therefore merely subjective inclusion is of no value, unless it is real inclusion, union or identity as well.

8. **Truth and Falsehood of Propositions.** A proposition may be true or false. A proposition is true when the object is in reality as it is said to be ; false when the object is different from what it is said to be. Truth admits of no degrees ; it affects the whole proposition. Error, on the contrary, admits of degrees, and a proposition may be more or less erroneous. This con-

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[1] How our cognitions are reduced to a perception of identity is explained by Balmes, in his *Fundamental Philosophy*, Vol. I., Book I., Chap. XXVI See also Joseph, *An Introduction to Logic*, p. 152.

formity of speech or thought with reality, called logical truth, is a property intrinsic to propositions. Hence some writers define propositions as statements of truth or falsehood.

Observe in passing the wrong opinion of those who define truth as the conformity of thoughts with other thoughts or with a full system of doctrine. Observe, again, that objects are true in themselves, according to their perfections; hence it follows that a proposition cannot at the same time be true and false, neither can it be true at one time and false at another. Once true always true, once false always false, applies equally to objects and to propositions. Truth is the same for all minds, i.e., it is universal and immutable, and independent of our wishes. The objects of our propositions may undergo a change in course of time, but this does not change the truth of a proposition which includes the time *in* predication as explained before.

The truth of a proposition is not to be confounded with the states of the mind called certitude and doubt, in which there are degrees that vary in the same person and regarding the same proposition. If someone asserts: " There is more pleasure in the study of Philosophy than in that of Rhetoric ", some will say yes, some no; and if I state that " the number of stars is even," nobody can either affirm or deny it, and yet it must be either true or false.

9. Quality and Quantity of Propositions. As regards quality, propositions are either affirmative or negative. The two forms are essentially different. In the affirmative the mind unites the predicate with the subject; in the negative it separates the predicate from the subject; and in both cases the mind is guided by the clear perception of what things are in reality. The separation is expressed by the words "no", "not", which are part of the copula, performing the function of the mental act of division between subject and predicate. A negative word attached to the terms belongs to the matter, as in the following: "Every man who does not sin is good"; where the copula is unaffected by the negative subject.

"Some logicians", says Bain, "have proposed to do away with the distinction between affirmative and negative by transferring the sign of negation from the copula to the predicate; "A is-not B", "A is not-B"; "penury is not agreeable", "penury is disagreeable." There is then the appearance, but only the appearance, of making all propositions affirmative. The attempt is illusory. Affirmation and Denial belong to the very nature of things; and the distinction, instead of being concealed or disguised to make an imaginary unity, should receive the utmost prominence that the forms of language can bestow." [1]

The quantity of a proposition means the extension of the subject we are speaking of, namely the whole or part of it. This adds a new form to the propositions:

All **S** is **P,** No **S** is **P,** Some **S** is **P,** Some **S** is not **P.**

A proposition is universal when the predicate is referred to the entire denotation of the subject; particular when referred to part of the denotation of the subject. The words all, none, any, every, etc., are signs of universality; while some, few, most, many, not a few, etc., are marks of particularity. The logical sign of universality is *All,* that of particularity is *Some.*

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[1] Bain, *Logic, Deduction,* p. 83.

The beginner should notice the precise meaning of the two words *all* and *some*. **All** means that we speak of each and every one of the whole extension of the subject; thus, "All trespassers will be prosecuted" means that "any trespasser is liable to prosecution." Such is the meaning of *all* as a sign of universality. There are other meanings in common language: thus in "all the ships were divided into groups of five for the battle," the word "all" is collective and singular; and in "not all the students fail", it is particular, "some students do not fail."

Some as a logical sign of particularity is essentially positive and vague; it means one or more, and may actually cover all. It expresses our ignorance as to the extent of the subject, except that it is *at least* one, and perhaps indefinitely more than one. If the subject is qualified by "most", it is also "some", but indefinitely more than half. Such is the meaning of the word "some" as a sign of a logical particular proposition. There are other meanings in common use. Usually we say "some" when we suppose less than half. It may also be taken collectively as in the following: "Some mistakes do not spell failure."

To avoid ambiguity logicians say that the word "All" means that the subject is *distributed*, and the word "some" means that the subject is *undistributed*.

A singular proposition has a singular term as subject; but practically it is universal because the predicate is referred to the whole denotation of the subject, as in the following: "The earth goes round the sun."

10. Indesignate Propositions. A proposition in which the subject has no mark of quantification is said to be indesignate. The logician finds the quantity by looking at the meaning. For instance, "Men are mortal" is universal on account of mortality being a property of man. Here we are bound to leave the form and study the subject-matter of the proposition, which requires some special knowledge.

It is well to notice two kinds of universality, namely physical and moral. If I say, "Bodies are heavy", the universality is based on a physical property of matter, and is without exception In the following: "Parents love their children; boys are playful"; the universality is moral, and grounded on deep natural instinct.

At times it is difficult to settle the quantity of a proposition because of our ignorance of the connection between the subject and the predicate. A generic or necessary proposition is universal on the ground that the predicate is part of the connotation or a property of the subject. Empirical or enumerative propositions depend for their quantity on the knowledge of instances. Thus "All reformers are hated, all cases of cancer are incurable" are universal, as far as our knowledge of the past goes, but give no warrant for the future; while necessary propositions are universal for the past, present and future. Thus the propositions: "Water finds its own level, plants need oxygen, no pair of parallel lines enclose space, prudent people are honest," are universal for all time. Observe in practice that:

1. Moral universalities, being subject to exceptions, are not strictly speaking universal, but are rather quantified by "most."

2. An abstract name as subject makes a proposition singular, as "Civilization is progressive"; its extension appears in the concrete, "All civilized people are progressive, or inclined to progress."

3. Secondary quantifications affecting the whole proposition may be transferred to the subject. One may say: "Students sometimes come late", or "Some students come late." Otherwise the quantification may simply modify the subject or predicate, for instance: "There are people who are always late."

4. Numerical propositions, or subjects expressed in numbers, are usually particular. Thus "Half of the passengers perished" means about half, or "Some." When the number is meant strictly, as for example: "Five of the nine persons accused have been found guilty", this is equivalent to: "Five of the accused are found guilty, four of the accused are not guilty", both propositions are universal.

5. When the subject is taken in a material, logical or grammatical supposition, the proposition is singular, as in the following: "Man is a substantive noun, man is a species, man is a word of three letters."

The two forms of quantity and quality give rise to the fourfold scheme of propositions, which for the sake of brevity are expressed by the letters **A, E, I, O.**

A stands for universal affirmative, All **S** is **P, S** a **P.**
E stands for universal negative, No **S** is **P, S** e **P.**
I stands for particular affirmative, Some **S** is **P, S** i **P.**
O stands for particular negative, Some **S** is not **P, S** o **P.**

The letters are derived from the Latin words *affirmo nego.*

11. Quantity of the Predicate. The extension of the predicate needs no sign of quantification, for it depends upon the quality of the proposition. Hence the general rule: the predicate of an affirmative proposition is particular or undistributed, and that of a negative proposition is universal or distributed. That this is so, may be explained by examples. When I say, " the earth is a planet " the extension of the predicate " planet " is not explicitly before the mind, but looking at it in denotation, it is clear that I predicate not the whole class of planets but indefinitely one of them. The meaning is " the earth is one of the planets." Similarly, the expression " Englishmen are Europeans " signifies that all Englishmen are included among the Europeans; in other words, we take of the class Europeans only as many as are English. Hence the predicate of an affirmative proposition is said to be *undistributed.* It may happen that subject and predicate are co-extensive, as in definitions or in singular predicates, but this is due to the character of the subject-matter, not to the strength of affirmation.

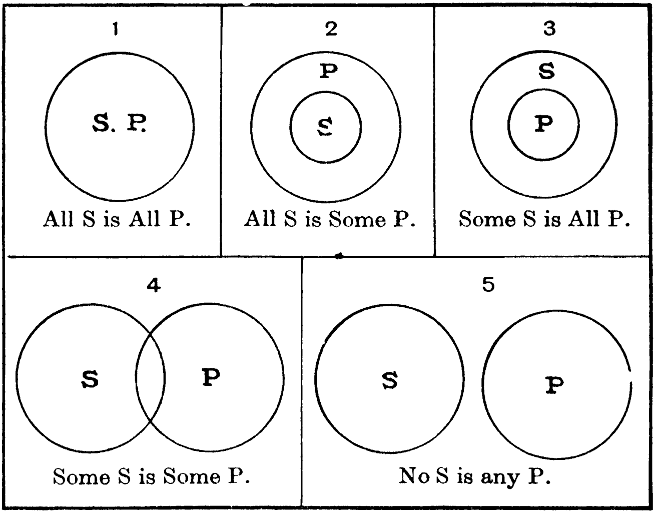
A negative proposition, on the contrary, separates from the subject the whole extension of the predicate. The proposition, " no books of Logic are novels " means that books of Logic are out of the entire class of novels, are none of the novels. The predicate, therefore, is *distributed* in a negative proposition. Summing up this exposition :

" **S** is **P** " means that **S** is included in the class **P**.
" **S** is not **P** " means that **S** is none of the class **P**.

Considering together the extension of both subject and predicate in the four traditional propositions, we see that :

A distributes the subject only.
E distributes both subject and predicate.
I distributes neither subject nor predicate.
O distributes the predicate only.

12. Diagrams. The relation of subject and predicate in extension may be instructively represented by geometrical figures. Of these Euler's system is the simplest. A circle is taken to stand for a class of things, that is, an enclosure of all the individuals of that class. The relations objectively considered are five :



All S is All P.

All S is Some P.

Some S is All P.

Some S is Some P.

No S is any P.

Observe that it is only the class-relation of terms that can be thus illustrated ; diagrams do not represent properly the logical propositions. They illustrate affirmation as inclusion, and negation as exclusive of classes of thing ; thought and logical rules cannot be substituted for by diagrams. Again, good diagrams should be self-evident, that is, one for each traditional proposition, and that capable of one interpretation—conditions which are more or less wanting in all the various systems. In the above figures "some", as indefinite, is not properly represented, and figure 4 may stand for **I** or **O** propositions.

13. The Import of Categorical Propositions. The import or interpretation of categorical propositions involves three main questions, namely, what it is that is referred to in a proposition; the implication of existence; and the precise relation between subject and predicate. Each point shall be considered separately.

(*a*) First let us settle what a proposition stands for, that is, whether it refers to language, to our ideas, or to relations of facts. This point calls to mind the different schools of logicians already referred to in the matter of terms, the nominalist, the idealist and the realist. Compressing the answer, we may say briefly that a proposition stands not for words, but for objects as conceived in the mind and expressed in language. We certainly examine the language of a proposition, but for the sake of thought; and again we look at the thought for the purpose of its truth in representing the objects. The meaning, however, before the mind, or what we aim at in our assertions is the expression of things as they are in themselves, or as we take them to be. Whoever says "the sun is up" certainly means that the real sun is above the horizon, although the statement has been shaped in the mind before coming down to an enunciation. The form of thought is unavoidable, and is more perceptible in abstract than in concrete propositions. The expression, "virtue is at times difficult," has an abstract notion as subject, yet in full analysis it conveys that "in reality there are acts of virtue which are difficult." In any form of expression it always comes about that common sense refers statements to reality.

(*b*) Another question is, whether a proposition implies the existence of the object referred to. A whole proposition refers to the same system of things to which the subject itself belongs ; whether present reality, or past history, or fiction, or abstract speculation, etc. Thus " Saturn has rings " refers to a real planet in astronomy, while " Saturn reigned over the golden age " refers only to an ancient myth. " Brutus killed Caesar " refers to real history, while " Hamlet saw his father's ghost " refers to dramatic fiction. " His blood cries for vengeance " refers to poetic symbolism ; while " virtue is strength " refers to abstract philosophy. Each of these propositions is true for a definite order of reality, or universe of discourse. As to the present question, it is only when dealing with a real subject, past or present, that a proposition can be said to imply real existence. The following rules may be laid down in this matter :

(1) A proposition having a concrete significant subject, singular or particular, must be taken to imply its existence, as when I say " Great Britain is an island ", " Some metals are liquids."

(2) If the subject is a universal common term, for example, " All experts are judges of scientific matters ", we do not assert the existence of its whole denotation ; and similarly when the subject is a notion or an abstract term, as in the following : " Pride will have its fall."

(3) Propositions in which necessity or impossibility is expressed do not involve the existence of either subject or predicate, because the whole stress is laid on the modality. For instance,

"Every event must have a cause" does not say that there is any particular event or any particular cause. Similarly, "Violent actions cannot last" implies no particular violent action.

(*c*) The third question proposed is under what aspect we regard the subject and the predicate of a proposition in connotation, or in denotation, or in both. There are four theories in which a proposition may be interpreted :

(1) Subject in denotation, predicate in connotation. In this sense the proposition expresses a relation of possession, for it means that the individuals denoted by the subject possess the attribute of the predicate—Predicative view.

(2) Subject in denotation, predicate also in denotation ; that is, the class of individuals of the subject are included in the class of individuals of the predicate, which is a relation of inclusion—Class view.

(3) Subject in connotation, predicate also in connotation ; that is, the attributes of the subject are accompanied by the attribute of the predicate, which means a relation of concomitance.

(4) Subject in connotation, predicate in denotation ; that is, the attributes of the subject indicate the presence of an object signified by the predicate, which is a relation of indication.

None of these interpretations can be called altogether wrong, as there are circumstances under which they can be used. Thus the fourth interpretation fits in with such a saying as "All that glitters is not gold." The third interpretation can have its use in strictly notional propositions, such as "Life is motion." But these usages are unusual and not according to our normal habits of thinking. The class view, which takes the terms as classes of objects, was defended by Sir W. Hamilton, whose theory caused a revolution for a time threatening to push other interpretations out of the field of Logic. Its main point is the quantification of the predicate. It is true that we do sometimes expressly wish to assign the subject to a class, as, for instance, when we say "Hindus are Aryans"; "Kunbis are cultivators"; "Jones is a merchant"; "Smith is an Englishman", etc., and in such cases we do quantify the predicate. But this is not true of the run of our predications.

In Hamilton's system the function of the proposition is to state an equality in denotation, thus "**S** is **P**" means x individuals of the class S are x individuals of the class **P**. Now such equality is to be rejected, for in actual predication what is before the mind is the attribute of the predicate, not the individuals. On the other hand, equality refers only to denotation, quantity or number, and takes no account of connotation; it lays no stress on what we are saying of the subject, nor justifies the distinction between subject and predicate. It may be said in favour of this system that it gives rise to four more propositions, namely: (v) toto-total affirmative—all **S** is all **P**; (x) parti-total affirmative—some **S** is all **P**; (y) toto-partial negative—no **S** is some **P**; (z) parti-partial negative—some **S** is not some **P**. But those propositions, in which the predicate is quantified against the traditional rules, are unnatural and (except perhaps " *u* ") never used, because our meaning can be better expressed in the usual forms **A, E, I, O.**[1]

The predicative view represents the natural habit of our minds, which is to describe, rather than to classify things. In this manner of interpreting the proposition the subject is read in denotation, for it is quantified, and also in connotation because it has a meaning ; but the predicate is taken in connotation only. Thus " **S** is **P** " implies that **S** has the quality **P**, and this interpretation, as we said before, is in accordance with the ordinary way in which we think. Except where we actually wish to classify, the quantification of the predicate is not before the mind in the very act of affirming or denying. This is clearly so whenever we predicate an attribute, or an attribute-like expression, for instance, " Students are liable to error ", or " His behaviour is not as it should be." Nor is it otherwise when the predicate is a substantive noun, for example : " Some politicians are not statesmen " ; for even here the qualities of a statesman are foremost in the mind, rather than individuals. Moreover, the nature of predication consists in asserting identity in difference between subject and predicate, and this relation is verified only by taking the predicate in connotation. In saying " These boys are playing ", we assert identity of the quality playing with the subject boys in the midst of other qualities which are left out of consideration. Finally the predicative view stands at the basis of the traditional four-fold scheme of propositions, and has a claim to be accepted as the universal way of interpreting categorical propositions. This view may be summarised in the following expressions :

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[1] For a full discussion see Keynes, *Formal Logic,* Chap. VII. ; Welton, *Manual,* Vol. I., pp. 200-9.

"**S** is **P**" means **S** is the thing **P** or includes the attributes of **P**.
"**S** is not **P**" means **S** is not the thing **P**, or excludes jointly the attributes of **P**.

14. How to Throw Sentences into Logical Form. A very useful exercise should be mentioned here, before treating of the various kinds of propositions. It consists in rendering a sentence into strict logical form ; bringing into clear light the assertion, so as to deal safely with its logical implications. In common language the parts of a sentence are often transposed, complex, ambiguous, expressed in poetical words and incomplete. Now Logic looks at the meaning, that is, at the thing spoken of and what is said. Hence the necessity of re-stating a sentence in the explicit form of subject, predicate quantity and quality. This may require some changes in the grammatical structure so as to make plain the true import of the proposition. The following indications will serve as a guide to the student :

(1) The first is to make clear what is the subject and what is the predicate in a given sentence. Notice that in grammar the subject is usually the principle noun and likewise the predicate is the principal verb, but the subject of a logical proposition is always what the writer speaks of, and the predicate is that which is affirmed or denied of the subject. For instance, "We know what the monsoon is from experience." Here the word "we" is not the logical subject. The logical proposition is as follows : our knowledge of the monsoon is derived from experience.

(2) The terms may contain many words, hence choose first the whole subject, add the copula and push the rest into the predicate. Example : " No man ever fails to remain poor, who is both ignorant and lazy." Log. form : No man both ignorant and lazy is ever one who fails to remain poor.

(3) The subject is usually more definitely determined in extension, while the predicate is more general and uppermost in intension. Example : " The best thing you can do for the people is to make them jolly." Log. form : To make people jolly is a good thing you can do for the people. Again : " Nothing is more humiliating than failure after boasting." Log. form : Failure after boasting is a thing most humiliating.

(4) The wording at times must be changed in order to express the meaning clearly and precisely. Example : " Facts are stubborn things." Log. form : All facts are things which cannot be altered by denying or opposing them. Again : " Pride will have a fall." Log. form : All proud people are liable to fall.

(5) If terms are omitted, complete the proposition. Examples : " Fire ! " Log. form : The premises are on fire.—" Wonderful ! " Log. form : That is wonderful. —" Thieves ! " Log. form : These men are thieves.

(6) Grammatical sentences other than statements, namely, those expressing wish, command, or interrogation, may be turned into logical propositions according to the truth contained in them. Examples : " What do you think of it ? " Log. form : Your opinion is asked about the matter.—" Go home." Log. form : That you go home is my command.—" Put not faith in tale-bearers." Log. form : Putting faith in tale-bearers is not advisable.

(7) There are statements which, for the sake of emphasis, are expressed with emotion or interrogation. Examples : " How poor are they that have no patience ! " Log. form : All who have no patience are poor people.—" How can you have civilization without morality ? " Log. form : Civilization without morality is not possible.—" What a stupid thing ! " Log. form : That thing is very stupid.

(8) Idioms or proverbs are vivid expressions of a familiar truth. When translated into logical propositions they become clearer, but lose their wit and power of imagination. Examples : " When the cat has something, she purrs." Log. form : Some people on becoming rich are insufferable.—" Put your shoulder to the wheel." Log. form : Earnest application to work is the thing needed.—" Put not thy hand between the bark and the tree." Log. form : Meddling in family affairs is not allowed.

Observe, finally, that we fix upon the logical subject, looking not to the form of words but to the underlying thought. This often depends on the intention of the writer and on the context. Take this statement : " I read your letter with great sorrow indeed." The attention may be fixed upon the fact that it was " I " who read the letter with sorrow, and then " I " is the logical subject. Or the attention may fall on the reading of the letter, and the meaning will be this : the feeling I experienced on reading your letter is one of great sorrow.

CHAPTER V

ANALYSIS OF PROPOSITIONS

1. Simple and Complex Propositions. The simple categorical proposition, as to its general character, has been studied in the previous chapter. We now pass on to the study of its various kinds. The simple categorical contains one subject and one predicate only ; the terms, however, may be modified in their meaning by clauses, adverbs and limitations of every description. When such modifications belong to the terms and not to the copula, they constitute a kind of complex proposition. To the simple statement, " Bombay is a city ", we may add circumstances of time and place, adverbs, adjectives or subordinate clauses, by which the proposition will become materially complex, although formally simple. For instance : " The city of Bombay, located on an island enlarged by reclamation, is the second city of the Empire in commerce and population." It is easy to see that the modifying touches enlarge the connotation of the simple terms to which they are applied, and thereby limit in most cases their extension, as in the following propositions : " A house *such as you want* is not easily to be found. Uneasy lies the head *that wears a crown*."

2. Modality of Proposition. In modal propositions the predicate is referred to the subject in a special manner. This particular manner, called modality, affects the copula and adds a new meaning to the proposition. Compare the meaning in the following pairs :

1. God is holy.
   It is necessary that God be holy.
2. Prices will rise next month.
   Prices may rise next month.
3. He met with an accident.
   He probably met with an accident.
4. The course of the sun is not stopped.
   To stop the course of the sun is impossible.

The modalities recognised in traditional Logic are **necessary, contingent, possible, impossible,** which in ordinary discourse may be expressed in a variety of ways. The verbs must, may, can, etc., and several adverbial expressions are used as signs of modality. As for logical forms, a modal proposition may be rendered in two ways, for instance, " The sun will certainly rise to-morrow ", or " That the sun will rise to-morrow is certain." In the latter form the proposition is always singular ; in the former the quantity depends upon the modality ; that is to say, *necessary* and *impossible* express universality, while *possible* and *contingent* express particularity.

Modality which affects the predication is to be distinguished from modality conveying certitude or doubt in the mind. The former is objective, the latter subjective, and both are used in ordinary speech. The sense of the proposition in a given context will make clear whether the modality is subjective or objective. If a doctor says : " My patient is probably consumptive, but I will investigate further," he expresses a subjective opinion. But a statement like this : " There may be an early monsoon next year " means objective probability, derived from the nature of the monsoon, which may happen one way or another irrespective of opinion.

Logical modality requires two conditions: (1) It must affect the relation between subject and predicate, whereas a *material* modality that merely affects the terms, e.g., " He is occasionally angry ", has no logical importance. (2) It is understood to be objective, for it is meant for the purpose of inference and truth, which always rests on reality. This *formal* modality gives rise to four categorical propositions :

S must be **P** ; **S** cannot be **P** ; **S** may be **P** ; **S** need not be **P**.

Modern writers after Kant have divided modal propositions into the **apodictic,** which asserts necessary agreement or disagreement ; **problematical,** which asserts possible agreement or disagreement ; and **assertive,** which affirms or denies a relation of facts of experience, but says nothing about the manner of their relation. Comparing the three together, we find that apodictic propositions express a necessary relation in connotation ; the problematical ones state that the attributes of the predicate may or may not be united to those of the subject ; and the assertive merely say *is*, *is not*, laying stress on the extension of the subject ; for instance, " All metals are conductors ; Most metals are solid ; Few men are experts ; Some candidates fail in the examinations." Thus we have three forms :

S must be **P** ; **S** may be **P** ; **S** is **P**.

This threefold division is acceptable, if the modality be understood in an objective sense. What is objectionable is the subjective view taken by Kant, and by certain modern writers after him. According to them, each of these propositions expresses only a state of the mind, or a different degree of assent in our belief. That is not the logical meaning of modality, which ought to represent the true objective state of things. If this objective relation is discarded we miss truth; and our knowledge remains, so to speak, up in the air. Besides, the proposition " **S** is **P** " has nothing to do with modality; it merely asserts existence ascertained as a fact without touching upon the manner of predication.[1] To repeat therefore: No matter whether a proposition be modal or not-modal, Logic abstracts from the states of the mind, and looks only at what reality forces the mind to hold as true. Every judgment, of its very nature, claims to be true, and is therefore apodictic, as far as our assertion goes; a problematical judgment in the subjective sense is rather a suspension of judgment.

3. Compound Propositions. A statement with two or more subjects or predicates is a compound proposition, as it logically involves two or more independent assertions. Examples:

1. "Thirty days have September, April, June and November." This statement contains four subjects, and consequently as many propositions.
2. "Britain has often been at war and has acquired foreign possessions." This implies two assertions about the same subject.
3. "Neither science nor money can prevent death." This is a double negative proposition with a common predicate.
4. "He is clever but not good." This contains two propositions about the same subject.

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[1] For further explanation consult Keynes, *Formal Logic*, pp. 84-92; Joyce, *Principles of Logic*, pp. 58-61; Welton, *Manual of Logic*, Vol. I., pp. 192-5; Ueberweg, *System of Logic*, pp. 206-14.

4. Exponible Propositions. There are propositions apparently simple, but in reality compound, including two statements; and as such they are called exponible, because they require exposition. The two most common kinds are the exclusive and exceptive. Examples:

Exclusive. Only members are admitted. This means
1. Some members are admitted (**I**).
2. No non-members are admitted (**E**), or All admitted are members (**A**).

Exceptive. All but a few are pleased. This means:
1. Some (most) are pleased (**I**).
2. Some (few) are not pleased (**O**).

Observe that in the above example the exclusive particle falls on the subject; but it may likewise affect the predicate thus:

The High Court decides only important cases. This means:
1. The High Court decides important cases (**A**).
2. The High Court does not decide unimportant cases (**E**).

An exclusive proposition with a negative copula is similarly resolved into two propositions, one affirmative, the other negative. Examples:

Foreign goods alone are not protected. This means:
1. Some foreign goods are not protected (**O**).
2. All not-foreign goods are protected (**A**).

If negation falls on the exclusive particle itself, the statement resolves into two affirmative propositions, thus:

"Man is not only intelligent," resolves into
1. Man is intelligent (**A**).
2. Man is something else besides intelligent (**A**).

Notice that of the two propositions implied in the exponible, the universal or the more general is to be preferred for the purpose of inference, as it contains more meaning than the other. For the rest, exclusive and exceptive propositions are in reality almost the same thing ; their only difference lies in the way of expressing the restriction.

We should mention other categorical propositions which imply a hidden compound in a condensed form. For instance :

(*a*) Propositions with a definite numerical subject : e.g., " Five of the accused have been found guilty " implies that the rest have been discharged.

(*b*) Inceptive and desistive propositions expressing beginning or ending, e.g., " Motor-cars came into use in the twentieth century " means that no motor-cars existed before. " Imperial Government ceased in Germany after the great war," implies that Imperial Government existed there before that war.

(*c*) Propositions suggesting an additional thought, for example : Books are not altogether dead things. He, being dead, yet speaketh. A professional man, as such, is bound to secrecy.

5. Analytical and Synthetical Propositions. A proposition in itself is always a synthesis of subject and predicate ; that is, the subject becomes determined to one meaning or another by the addition of the predicate. The division between analytical and synthetical is based on the reason or motive why the predicate is attributed to the subject. When the reason of predication is found in the analysis of the contents of both ideas, then the proposition is said to be analytical. For instance :

" The whole is greater than its part " is seen to be true by the analysis of what a whole and a part is.

Synthetical propositions are those in which the agreement or disagreement between the subject and predicate is not learnt by the connotation of the terms themselves, but from another source, namely, experience or authority. In the following : " Mercury is liquid ; lead is heavier than iron ; horned animals are ruminants," the predicate is known to agree with the subject only by experience or study of the objects. That London and New York are the largest cities of the world is a fact generally known from authority.

Analytical propositions are absolute or independent of any conditions ; universal, and also self-contained, because the predicate is found in the conception of the subject and is merely drawn out of it. **Synthetical** propositions, on the contrary, are often not universal, and are not self-contained, because the predicate is not found in the concept of the subject, but has to be derived from some other outside source.

These two kinds of propositions have assumed in modern times the names of **verbal** and **real** propositions, meaning that in the first the predicate is found in the subject, consequently the same thing is expressed twice in different words. In the second the predicate comes from reality or experience. They are also called respectively **explicative and ampliative,** since the analytical proposition merely *draws out* or *unfolds* the meaning of the subject, while the synthetical *enlarges* its meaning.

The same division coincides with that of **a priori** and **a posteriori** propositions, in the sense that the motives or evidences for asserting the proposition are prior to, or posterior to experience.

Observe that analytical and synthetical propositions, (or in other words, verbal and real predications), are clearly connected with the predicables and also with the doctrine of definition. Real or synthetical propositions, generally speaking, predicate attributes that are not implied in the connotation of the subject, but are merely properties and accidents. On the contrary, in verbal or analytical propositions we refer to the subject what is included in it, as genus, or difference, or species.

Observe again that with the constant growth of the experimental methods, not only propositions, but sciences also have come to be classified as *a priori* or *a posteriori*, analytical or synthetical. Mathematics is called an *a priori* science, because it reasons from the notions of space and time ; History is *a posteriori*, as it is wholly dependent on facts of experience.

The above division seems clear and complete, embracing all propositions ; yet Kant devised a third kind. He maintains that all analytical propositions are *a priori*, but not all synthetical are *a posteriori* ; there are, according to him, propositions which are synthetical and *a priori*, which are neither based on the analysis of the notions themselves, nor depend on experience, but on a pure subjective activity of the mind. Examples : " Every event has a cause ; A straight line is the shortest distance between two points ; Two parallel lines cannot meet ; Matter and energy remain always the same ; The world had a beginning ; In all physical motion, action and reaction are equal ", etc. On careful inspection, however, it will be found that in all cases the judgment depends either on experience or on the analysis of the subject ; so that the proposition is either simply *a priori* or simply *a posteriori*.

The following refutation of Kant's reasoning will be useful :

" Kant adds ", says Palmes, " that this proposition : ' A straight line is the shortest distance between two points,' is not purely analytic, because the idea of *shortest distance* is not contained in the idea of *straight line.* Waiving the demonstration which some authors give, or pretend to give, of this proposition, we shall confine ourselves to Kant's reasons. He forgets that here the straight line is not taken *alone,* but compared with other lines. The idea of straight line alone neither does nor can contain the idea of *more* or *less* ; for these ideas suppose a comparison. But from the moment the straight line and the curve are compared with respect to *length,* the relation of superiority of the curve over the straight line is seen. The proposition is then the result of the comparison of two purely analytic conceptions with a third, which is *length.* If Kant's reasoning were good, even this judgment, ' The whole is greater than its part ', would not be analytic ; for the idea of *greater* enters not into the conception of the *whole* until the *whole* is compared with its *part.* Thus the judgment : ' Four is greater than three,' would not be analytic, because the idea of *four* until compared with *three* does not include the conception of *greater.*" [1]

6. The Hypothetical Proposition. It consists of a pair of categorical propositions connected by the copula " if ", which makes one of them dependent on the other. The part dominated by the " if " is called the Antecedent ; while the other, which depends on it, is called the Consequent. Thus, **if** A is B, **then** C is D. The drift or implication of this proposition has a peculiarity of its own, which consists in saying that the consequent " C is D " necessarily follows from the antecedent " A is B " which stands as a ground for predicating " D " of " C " in the consequent. As the fulfilment of the antecedent is a condition for the realisation of the consequent, it can truly be said that hypotheticals are conditional propositions. The truth of such propositions depends upon the truth of the connection between the two categoricals, not upon the truth of each one separately. The following : " If this college had not existed, I should never have been enrolled in it ", is true ; but the separate statements, if put into categorical form, would be false. Again both statements may be true and the hypothetical connection a fiction, for instance : " If it is eleven o'clock, it is raining." The hypotheticals admit quantity and quality ; and in order to ascertain them the relation must be looked to, namely, whether it states that the consequent follows or does not follow from the antecedent, and whether it states that the consequent follows universally or particularly. In common language, quantity and quality are not only attached indifferently to the antecedent or consequent, but they, as well as the copula " if ", are variously expressed. Examples : " The meeting will be held, provided the weather is fair."—If the weather is fair the meeting will be held. Again : " Unless it rains, misery is likely to be felt."—Sometimes if there be no rain, misery is felt. Another example : " Generally speaking, you will rule over people better by commanding your passion than by force of arms."—Many a time if you command your passions, you will rule over people better than by force of arms.

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[1] For a masterly criticism of Kant's synthetic *a priori* judgments see Balmes, *Fundamental Philosophy*, Vol. I., Chap. XXIX.

We are now in a position to compare the hypothetical proposition with the categorical and to notice how far they differ from each other. In the categorical proposition the predicate is referred to the subject as actually belonging or not belonging to it. In other words, a categorical statement is concerned with facts as they are, but does not declare the relation that exists between subject and predicate. For instance, " The students are sitting down " states the agreement of the quality " sitting " with " students ", but how the quality comes to be there, the proposition does not say. The hypothetical proposition, on the other hand, expresses something different, namely, that a certain fact depends upon another. For instance, " If the weather is good the meeting will be held." In this sentence there is no statement of any fact, but what we state is that the meeting depends upon the weather. In this respect the hypothetical proposition is said to be more scientific than the categorical, since it refers to some kind of causality, order, or system of things. What makes particularly for our purpose here is the remark that some categorical propositions approach nearer to the hypothetical form than others. We may distinguish two kinds of categorical propositions, the Generic and the Empirical. In the generic there is a hidden relation of dependence, in the sense that the subject contains the reason of the predicate. Thus, " Prudent people are pleasant " indicates that people being prudent are pleasant. Again, " Educated persons are progressive " means that education in a person is a source of progress, and " Heated metals are expanded " amounts to this, that by heating we cause expansion. Such propositions are easily converted into the hypothetical form. Equally well, may one say: "If people are prudent, they are pleasant; if persons are educated, they are inclined to progress; if metals are heated, they expand." The empirical proposition suggests no relation beyond the fact of agreement or disagreement. For instance, "The days of the week are seven," or "The months of July and August are rainy in the tropics," cr "Plants are green", are mere statements of facts. Propositions like these do not lend themselves to the hypothetical form. The generic proposition stands between the empirical and the hypothetical. We can say in general that the distinction between the categorical and the hypothetical propositions marks a degree in the way of looking at things. Our knowledge begins by singular facts, which are expressed in categorical propositions, and on further inquiry, we come to know that one fact depends upon another, and this gives rise to a hypothetical proposition.[1]

7. Disjunctive Propositions. A disjunctive proposition affirms one or another of two or more predicates; that is to say, it expresses an alternate predication by means of the particles *either, or*. When the disjunction is applied to the subject it means a predication of alternative subjects; hence the two forms:

A is either B **or** C; **Either** A **or** B is C.

Regarding the import of disjunctive propositions there is a variety of opinion. The ideal or perfect disjunction requires that the alternate predicates be exhaustive and exclusive. It is admitted, however, that in common language the predicates need not be exclusive. The implication necessary and sufficient is: "If it is not one, then it is the other", as in the following: "He is either timid or very modest"; he may be both, but at least "If not one, the other". The truth, therefore, of the proposition consists in the fact that the matter referred to in the proposition is in reality exhausted by the alternate predication according to the circumstances known to the speaker. When I say, "He is either in Poona or in Bombay", I mean that at present he is not in any other place.

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[1] For a full discussion, see Bosanquet's *Logic*, Vol. I., pp. 207-13.

The other implication, namely, "If it is one, then it is not the other" belongs to a perfect disjunctive which is both exhaustive and exclusive; for instance, "Either he works for his country, or his patriotism is very low." Therefore, in cases like this, a disjunctive proposition has a double implication.[1]

As to quality, such propositions are always affirmative. The expression, "A is neither B nor C", is not a disjunctive. It is a compound proposition stating what A is not. Again, their quantity is always universal. We may indeed introduce a quantity of particularity by saying: "Some A is either B or C", but such a proposition has no logical value.

We should mention here a kind of conjunctive proposition expressing the incompatibility of two facts, e.g., "You cannot keep your cake and eat it", or in general form: "**Not both** A and B." The implication of this proposition is "If one, then not the other."

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[1] Some logicians would require all disjunctive propositions to be exclusive alternatives, as held by Fowler, *Distinctive Logic*, p. 119; Bosanquet, *Logic*, Vol. I, p. 324; Bradley, *Principles of Logic*, Vol. I, Chap. IV. Many others maintain that a disjunctive form need not imply exclusive alternatives. See Welton, *Manual*, Vol. I., pp. 187-90; Keynes, *Formal Logic*, pp. 277-81.

8. **Exchange of Relation in Propositions.** Disjunctive propositions are similar to the hypothetical. Both kinds are conditional, since both place a limitation on the fulfilment of the categorical statement. Hence one form is easily transformed into the other. Again, both may be modified into categorical; but the process involves a change of relation, and consequently some meaning is lost. For instance: Either Socrates was an enemy of religion, or the Athenians were unjust in putting him to death.—If Socrates was no enemy of religion, the Athenians were unjust in putting him to death.—Socrates, being no enemy of religion, was unjustly put to death by the Athenians.

The conjunctive proposition expressing incompatibility may likewise be changed, whereby we arrive at certain equivalent forms useful for the purpose of inference. Let **S** and **P** stand for propositions or facts, and we have the following equivalent expressions:

Everything is either **P** or not-**S**; Nothing is both **S** and not-**P**; If anything is **S**, it is **P**; Every **S** is **P**.

CHAPTER VI

IMMEDIATE INFERENCE

1. What a Law of Thought is. The term law has various meanings, but it always implies an established uniformity that regulates action. The expression of such uniformity is the statement of a law; as when I say: "All bodies are subject to gravitation; Heat expands bodies; Friction produces heat." A law is called a principle because it contains the ground for other regularities or concrete facts, so that we can draw conclusions from it.

Every science has its principles, even **first** principles upon which the whole edifice rests as on its foundations. We are concerned here with the fundamental principles of Logic, which are not restricted like those of other sciences. Logic regulates thought in any subject-matter; hence its fundamental principles are most universal and extend to all possible knowledge. For this reason they are called laws of thought. They are also **final,** because it is impossible to go beyond them; **axioms,** because they are universally received and self-evident; **postulates** of knowledge, because they are assumed and taken for granted.

2. The Need of Laws of Thought. Logic deals with forms of thought; that is, with the ways in which we think. We have already seen some of these forms when laying down the traditional propositions **A, E, I, O,** etc. But we need more than a mere acquaintance with the various forms; we are concerned with correct thinking. Now the laws of thought serve as a basis for the validity of the forms of thought, to the correct use of them, and to the consistency of the thought-process as a whole. Our inferences and trains of reasoning are tested by, and finally traced back to those primary and unshakable foundations. The fundamental laws of thought have the force of nature, so to speak, to impose themselves upon our thinking processes, as determining their validity.

The traditional laws of thought are four: the Principle of Contradiction; the Principle of Excluded Middle; the Principle of Identity; the Principle of Causation.

3. **The Principle of Contradiction** is expressed thus: **Nothing can at the same time be and not be** (exist and not-exist), or: **It is impossible at the same time to affirm and to deny.** In symbols: **A cannot at the same time be A and not-A,** or, **I cannot say: A is X and A is not X.** The first expression refers to reality of things or to existence; the latter to affirmation or negation in propositions. In other words, the first expresses a law of being, the second a law of consistency in thought.

The essential meaning in this principle is the absolute exclusion of being by not-being, and of not-being by being; in short, incompatibility between the two. This is an axiom, a truth, a fact of immediate evidence, an object of clear intuition, as any one can experience for himself. The consequence is that two contradictories cannot be true; if one is true the other is false ; an affirmation implies a negation of the contradictory.

The conditions necessary for this principle are that the contradictory terms be understood in the same sense, in the same respect and at the same time ; otherwise the law does not hold good. A man may be wise and not-wise in different matters, say politics and mathematics ; and an object may be white and not-white on different sides, etc.

4. The Principle of Excluded Middle. This law is a supplement or extension of the law of contradiction. It is the previous principle looked at from another point of view. It is expressed thus : "**Everything must either be or not-be,**" or : "**Everything must be either X or not-X.**" The meaning is that between two contradictories there is no middle term ; in other words "A and not-A exhaust the whole field of reality and of thought." The consequence is that if one is false the other is true ; a negation implies an affirmation of the contradictory. The principle of contradiction states that two contradictories cannot be true together ; whereas the principle of excluded middle states that two contradictories cannot be false together ; and this is precisely the difference between them.

The first condition of this principle is that the two alternatives be really contradictories, or else the axiom fails. Thus holy, unholy ; easy, uneasy ; hot, cold ; friend, enemy, are not contradictories but contraries, and both may be false together. It is on this principle that "two negatives make a positive." Hence the expression, "it is not unlikely" means "it is likely." Moreover, there is a vast distance between what the law of excluded middle—and for that matter the law of contradiction—is in itself and what it is to us. Two contradictory propositions are so related that one must be true and the other false, but which is which is not given us to know on the strength of the law. Take these propositions: "Mars is inhabited, Mars is not inhabited." As far as our knowledge goes, both are neither true nor false, but in themselves one is true and the other is false.

5. **The Principle of Identity** is expressed in this way: **Everything is what it is,** or **A is A**. The principle is not tautological or a mere repetition; it means identity amidst some diversity. We shall point out three of the various meanings given to it by different authors:

(1) **A is A** expresses the unambiguity of the act of judgment; that is to say, truth is something fixed; or the contents of a judgment are invariable, or independent of changes of time, space and person. In this sense, they say, the Law of Identity is a fundamental principle of Logic; for if the contents of a judgment could be variable, all thought and all reasoning would be chaotic. Hence a statement once true is always true; once false is always false.[1] This interpretation is open to objections: First it does not differ from the principle of contradiction; for to say: "A judgment once true is always true" is to say: "A true judgment cannot be false." Secondly, to describe **A is A** merely as "continuity of truth or falsehood" leaves out the note of "identity amidst diversity," and deprives the principle of its most important meaning.

(2) Mill gives the following enunciation of the Law of Identity: "Whatever is true in one form of words,

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[1] Bradley, *Principles of Logic*, p. 143.

is true in every other form of words which conveys the same meaning"; a postulate (he says) which it is necessary to make in connection with the use of language, since we must be free to express our thoughts in a variety of words.[1] The objection to this view is that Mill's postulate refers to the use of language, not to exactness of thought.

(3) The third meaning is that adopted by traditional writers on philosophy. "**A is A**" means that every thing is identical with all its attributes and perfections. For instance, "Man is mortal" means that mortality is an attribute of man. This way of explaining the principle makes it the basis of all our judgments, and appears to be the sound one. "If in all judgments there is affirmation of identity or non-identity, and all our cognitions either begin or end in a judgment, it would seem that they all ought to be reduced to a simple perception of identity. The general formula of our cognitions will then be: **A is A, or a thing is itself.** This result strikes one as an extravagant paradox, and is so, or not, according to the sense in which it is understood; but if rightly explained, it may be admitted as a truth, and a very simple one." [2]

6. The Principle of Causation. This law is also called the principle of sufficient reason. It may be expressed thus: (1) **Every event must have a cause.** (2) **Every statement that is true must have a sufficient reason for its truth.** Explanation:

(1) **Cause and event.** The principle refers to events,

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[1] Mill, *An Examination of Sir William Hamilton's Philosophy*, p. 482.

[2] Balmes, *Fundamental Philosophy*, Vol. I., p. 176. This author devotes chapters 26 and 27 to working out this point.

not to the ultimate cause, eternal and self-existing. Now an event is a thing that begins and was not before. This beginning cannot be due to the thing itself, because a thing which does not exist is nothing, and therefore cannot produce anything. It must have come into existence by the agency of something else, and that something else is its **cause.** Likewise every change in a thing which exists has also its beginning, and this beginning must be due to a cause, other than the change itself, which is rather an effect.

(2) **Sufficient reason.** The cause of our thought is our own mind working on the object present to it ; but when we go through a process of thought (for instance by making an inference) we require not only a cause of the process (which is the activity of our own mind), but a reason *why* the process should be exactly what it is. The sufficient reason will be either the *principles* of Contradiction, Excluded Middle and Identity (in the formal processes), or else the evidence of facts which justify the conclusion. By pointing out these principles or facts, we give a **sufficient reason** for our inferences. On the contrary, an inference will be either false or precarious unless we can point out some such sufficient reason for it.

The purpose of this law or principle in Logic is not to discover causes or reasons for things that happen (this belongs to the sciences) ; but to direct and check the soundness of our processes of thought, and so vindicate our knowledge.[1]

7. Questions about These Laws.—Various questions are discussed in books of Logic concerning the four traditional laws of thought:

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[1] For further explanation of the Laws of Thought, their import, application and history, consult Hamilton's *Lectures on Logic*, Lect. V. and VI. ; Ueberweg, *System of Logic*, pp. 231-86.

The first is their proper place in Logic. Some, Welton for instance, prefer to treat them before the concept, on account of their universal application. Others would place them at the beginning of propositions, while others find need of them in the process of inference.

Another question is their mutual relation. The three laws that make for consistency of thought are inter-related, as may be shown from the equivalent propositions: Every **S** is **P**; Nothing is both **S** and not-**P**; Everything is **P** or not-**S**. Writing **S** instead of **P**, we have the three laws as follows:

(1) Every **S** is **S**; the law of identity.

(2) Nothing is both **S** and not-**S**; the law of contradiction.

(3) Everything is **S** or not-**S**; the law of excluded middle.

Although they are inter-related, one is not proved by another; all are fundamental and rest on their own evidence.

The Law of Causation, in the sense that every event must have a cause, is connected with that of contradiction, as manifested above; but in the sense of a general principle of knowledge or of sufficient reason, it is fundamental and distinct from the others.

Which of these laws is first or more fundamental; whether they are true both in the real and the ideal order; that there are these four only; and other similar questions may be studied in the standard books.

8. Positive and Negative Proof. The real function of these laws, and of axioms in general, is to be the foundation of all proof. An axiom, self-evident and incapable of demonstration, is **a principle, a ground** and **a reason.** Principle means a statement or a truth from which other statements or truths are derived. Another name for logical principles is Ground or Reason. Ground means basis, foundation, support. Hence, a logical ground means the logical or rational foundation of a statement, belief or conclusion ; in other words, that which logically implies the conclusion. Thus a truth is a principle, because from it other truths are derived ; or a ground, because by it other truths are supported ; or a reason, because through it other truths are shown to be true.

A proof means deriving one statement from another. A proof may be positive or negative. It is positive when we prove a thing directly, by showing from principles that it is so. We prove negatively by demonstrating that a thing is such on account of the impossibility that it should be otherwise. For instance, the proposition : " The exterior angle of a triangle is greater than either of the interior and opposite angles ", may be proved directly by a course of argument, or indirectly by showing the absurdity which follows from the supposition of its being equal to or less than either of them. Positive argument is better than negative, because the positive gives the ground. The negative does not show the immediate reason, but both are based ultimately on a first principle.

9. Immediate Inference. As we made a study of terms for the sake of propositions, so in like manner the analysis of propositions is for the purpose of inference, which is the last stage of our mental activities. In every process of inference we may distinguish three elements : the known proposition we start with, called Antecedent or Premise ; the statement arrived at, called Consequent or Conclusion ; and the connection between the two, which in the most appropriate sense of the word is called Inference. Now in the act of inference the mind sees the connection of one proposition with another, and naturally passes on to a new proposition. In this process, therefore, the mind passes from one statement to another. To make this somewhat clearer, observe that facts of experience are not an inference. If, on the evidence of facts, I make assertions like these : " There are soldiers in the street, there are horses in the street ", I am sure of both propositions and I may pass from one to the other, but this mental activity is not what is meant by inference. By inference is meant to pass from what is seen to that which is not seen ; from what is given to something else not given. Thus we have an inference, when from the smoke which is visible one comes to know the presence of fire which is not visible. Observe, again, that an inference is said to be " correct " or " incorrect ", and the conclusion, being a proposition, may be " true " or " false." For the rest, inference is divided into mediate and immediate, and each may be based either on the form of the propositions or on the matter itself. We shall first take up the usual kinds of immediate inference.

To repeat then, immediate inference is the process of deriving one proposition from another, or we may say, of unfolding the implication of a proposition. This mental activity is carried out in a great variety of ways. Thus, from the import of a disjunctive proposition we pass to the hypothetical form and vice versa, as explained before. When two propositions are logically opposed, we know the truth of the one from the falsity of the other. From an affirmative we can logically deduce a negation, and so on.

It has been questioned whether these processes that go by the name of immediate inferences have a claim to real inference. To quote: "It seems better to regard these processes as acts of verbal interpretation, or explanation of the meaning of propositions, rather than as inferences in the true sense of the word. They render important service in helping us to understand what is implied or involved in the propositions, but they do not lead the mind on to any new truth."[1] This controverted point is not perhaps of very great importance, but serves our purpose in throwing light upon what the act of inference is supposed to be. What is the precise meaning we should take of real inference? It may be summed up in two characteristic features: first the conclusion must differ from the premise. To say, for instance, "Monday is the second day of the week, therefore the second day of the week is Monday", is not an act of inference, for these propositions are identical. Both terms being singular, it makes no difference which is subject and which is predicate. Secondly, it is essential to inference that the propositions be connected with each other. If I say: "It is eleven o'clock, therefore it rains", I make a conclusion without inference, that is, without a ground on the strength of which I can pass from one statement to the other. Looking at each process of immediate inference, we can see to what extent the characteristics just laid down are verified. Neither the reasoning involved, nor the amount of difference between premise and conclusion are the same in these processes. But it is not difficult to see that in all of them—looking not at this or that example but at the process as such, there is more than a mere grammatical transformation of a proposition, for we pass from one statement to another not identical with the first. The conclusion, as a rule, is not a new truth altogether, but it contains an explicit statement not given in the premise, and affords a new aspect of the given truth. Moreover, the reasoning involved is correct or incorrect, not on account of grammatical rules being observed or not observed, but by reference to the laws of thought or to logical principles. To conclude, the name of immediate inference may be reasonably maintained for the usual processes traditionally known by this name. We proceed to define and to illustrate the various kinds of inference based on opposition of propositions, and those which consist in unfolding the meanings involved in the form or in the matter of propositions.

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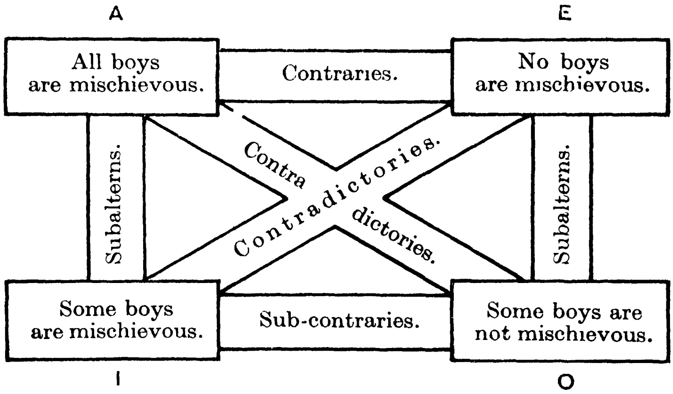
[1] See *An Introductory Logic*, by J. E. Creighton, p. 94.

10. Opposition of Categorical Propositions. By opposition of propositions we mean the relation that exists between two propositions having the same subject and predicate, but different in quantity or quality or both. **Subalternation** exists between a universal and a particular of the same quality; **Contradiction** between a universal and a particular of different quality; **Contrariety** between two universals of different quality; **Sub-contrariety** between two particulars of different quality. The square of opposition shows them all at a glance :


A
E
All boys are mischievous.
Contraries.
No boys are mischievous.
Subalterns.
Contra dictories.
Contradictories.
Subalterns.
Some boys are mischievous.
Sub-contraries.
Some boys are not mischievous.
I
O

The opposition that exists between two propositions which differ in quantity or in quality, or in both, gradually decreases from contradiction, which is the most important of all, down to subalternation. Contradictory propositions are strictly opposed, as they can neither be true nor false together ; contrary propositions are opposed only in regard to truth ; next come the subcontraries, that cannot be false together ; the subaltern propositions are barely opposed, since both may be true or false.

Observe carefully the precise distinction between contrary and contradictory opposites. From the square of opposition we know that a universal affirmative proposition is contradicted by a particular negative, and that its contrary is a universal negative. This is very simple, but not enough to put in a clear light what is meant by contradiction and contrariety.

Contradiction implies that one proposition affirms or denies precisely what is necessary to falsify the other and no more. In other words, two propositions are contradictory when one denies the least possible to break down the truth of the other, so that no middle can be found between the two. The great logical value of this kind of opposition consists in that we know the truth of a proposition from the falsity of the other, and vice versa, from the truth of one we can assert the falsity of the other. Besides, every proposition has its contradictory and only one, while singular propositions admit only the contradictory opposite. Examples: The propositions " this house is mine " and " this house is not mine " are strictly opposed, because one denies the least possible to falsify the other. Again: the proposition, " most employees are honest " is contradicted by " not most employees are honest," and likewise the proposition " two-thirds of the candidates failed " is contradicted by " not two-thirds of the candidates failed." Generally speaking, a proposition is strictly opposed by the expression " it is not true," or simply by the particle " no."

On the other hand, two propositions are contrary to each other whenever the one denies more than is necessary to falsify the other. The result is that such propositions are opposed in regard to truth, but both may be false together, because there is a middle between the two. Besides, while singular propositions have no formal contrary opposite, other propositions admit more than one. Thus going back to a previous example, the proposition " most employees are honest " is opposed by two contraries, namely, " no employees are honest " and " hardly any employee is honest ", for in either case we say more than is necessary to deny the original statement.

The distinction just drawn between contrary and contradictory opposites brings us to a practical remark. It is far better to oppose a general statement by the contradictory than by its contrary opposite. For instance the proposition, " all people are superstitious " can be easily overthrown by proving with a single example that " some people are not superstitious." To assume the contrary may not advance the cause of truth, for two contrary propositions may be false together ; and moreover, it is always a difficult task to establish a universal proposition concerning matter of fact.

Inference from Opposition. The inference from these pairs of oppositions consists in this : that from the truth of one proposition we draw the truth or falsehood of the other, and vice versa. Each pair has its proper law of inference.

Subalternation. From the truth of the universal we infer the truth of the particular, and from the falsehood of the particular we infer the falsehood of the universal ; not vice versa. **Proof** : The universal contains the particular ; hence if the universal is true the particular must be true, and if the particular is false the universal (in which the falsehood of the particular is included), must be false. Thus for example : If it is true that all men are mortal, it will also be true that some men (who are included in the denotation of the subject), are mortal. And if it is false that some men are liars, *a fortiori* it will be false that all men are liars.

We said " not vice versa." For, if it is false to say that all men are sincere, it may be true that some men are sincere. For it often happens that the predicate is not applicable to the whole denotation of the subject, but is applicable to part of it. Again, what is true of part of the denotation of the subject may not be true, (and usually is not true), of the whole denotation of that subject.

Contradiction. The law is that contradictory propositions cannot be true together, nor false together. **Proof :**

(1) Not true together ; for if the predicate is applicable to every member of the class that forms the subject, it must be false that it is not applicable to some members of the same. If all bodies are heavy, it must be false that some of them are not heavy, otherwise a portion of the subject would at the same time be heavy and not-heavy—a thing impossible, according to the Principle of Contradiction.

(2) Not false together ; for if it is false that the predicate " mischievous " is applicable to all the members of the class " boys ", it must be true that some are not mischievous ; otherwise there would be some boys who are neither mischievous nor not-mischievous, and that is impossible according to the Principle of Excluded Middle, namely, that a given object must be either one or the other of two contradictories. Hence from the truth of a proposition we infer the falsehood of its contradictory, and vice versa.

Contrariety. Contrary propositions cannot be true together, but may be false together. **Proof :**

(1) Not true together ; for if the predicate " mischievous " is true of every boy, it cannot be false of any boy—according to the Principle of Contradiction.

(2) May be false together ; for there is a middle

term between the two. It may happen that the predicate is true of some individuals of the subject and false of others. Hence from the truth of any proposition we infer the falsity of the contrary, but not vice versa.

Subcontrariety. Sub-contrary propositions cannot be false together, but may be true together. **Proof :**

(1) Not false together ; for if it is false that Some planets are stars, the contradictory, " No planets are stars " must be true ; and consequently, some planets are not stars. Let us take **I** and **O**. If **I** is false, then **E** is true ; hence **O** is true, and therefore **I** and **O** cannot be false together.

(2) May be true together ; for if I say : " Some boys are mischievous, some boys are not mischievous," I may refer the predicate to different portions of the class " boys." Hence if one of two subcontraries is false, the other must be true, but not vice versa—for both may be true.

The inferences from opposition give the following results :

If **A** be true,	**E** is false,	**I** is true,	**O** is false.
If **E** be true,	**A** is false,	**I** is false,	**O** is true.
If **I** be true,	**A** is unknown,	**E** is false,	**O** is unknown.
If **O** be true,	**A** is false,	**E** is unknown,	**I** is unknown.
If **A** be false,	**E** is unknown,	**I** is unknown,	**O** is true.
If **E** be false,	**A** is unknown,	**I** is true,	**O** is unknown.
If **I** be false,	**A** is false,	**E** is true,	**O** is true.
If **O** be false,	**A** is true,	**E** is false,	**I** is true.

11. Opposition of Hypotheticals and Disjunctives, etc. Hypothetical propositions are opposed in the same way as the categorical, for they have quantity and quality.

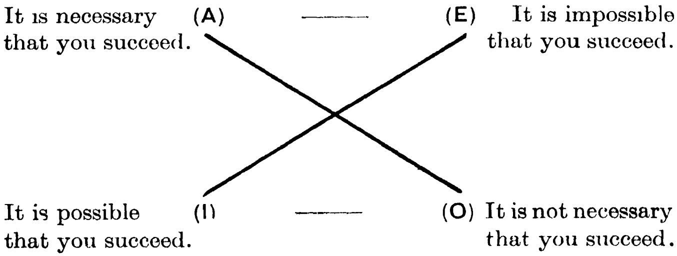
As an example note the following :

A. If P is Q, it is always R or it must be R.
E. If P is Q, it is never R or it cannot be R.
I. If P is Q, it is sometimes R or it may be R.
O. If P is Q, it is not always R, or it need not be R.

By using these or similar forms we can have the various kinds of opposition.

Disjunctive propositions, being universal and affirmative by their nature, are not opposed by another disjunctive. For instance, the imperfect disjunctive : " This is either certain or at least probable ", is contradicted by the double negative : " This is neither certain nor probable." And the perfect disjunctive : " You will either pass or fail in the examination," (or in the conjunctive form : " You cannot both pass and fail ") is contradicted by the double affirmative : " You can both pass and fail."

Modal propositions lend themselves to four kinds of opposition, based on the modalities necessary, contingent, possible, impossible, thus :


It is necessary that you succeed.
(A)
(E)
It is impossible that you succeed.
It is possible that you succeed.
(I)
(O)
It is not necessary that you succeed.

Similarly are opposed propositions which refer to space, as: Everywhere (**A**); Somewhere (**I**); Nowhere (**E**); Not everywhere (**O**); and those which refer to time, as Always (**A**); Sometimes (**I**); Never (**E**); Not always (**O**).

It is very important at times, for the sake of truth, to ascertain the precise contradictory of a given proposition. This is easy to do in simple propositions; but in complex and compound ones, it requires careful attention to every affirmation and negation, and to the intention of the speaker.

12. What Eduction is. Eduction takes place, when, from a given proposition accepted as true, we derive another whose truth is implied in it. It is done by reversing the terms negative or positive, or by changing their places; by which the quality is changed accordingly. The fundamental processes are obversion and conversion, the other forms of eduction being a combination of these two.

Obversion is an inference by which the new proposition has the same subject, but the predicate is the contradictory to that of the first proposition.[1] **Rule**: Replace the predicate by its contradictory, and change the quality of the proposition. Reason: Because the affirmation of a predicate implies the denial of its contradictory, and vice versa. Thus by saying S is P it is implied that S cannot be not-P, otherwise S could be P and not-P at the same time—a fact impossible according to the Principle of Contradiction. And No S is P implies that all S is not-P, on the principle of Excluded Middle.

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[1] Obversion has been called *Permutation*, *Equipollence*, *Immediate Inference by Privative Conception, etc.*

The forms of obversion, therefore, will be as follows :

Obvertend.	Obverse.
(**A**) All S is P.	(**E**) No S is not-P.
(**E**) No S is P.	(**A**) All S is not-P.
(**I**) Some S is P.	(**O**) Some S is not not-P.
(**O**) Some S is not P.	(**I**) Some S is not-P.
(**A**) All men are mortal.	(**E**) No men are immortal.
(**E**) No plants are sensitive.	(**A**) All plants are insensitive.
(**I**) Some men are fallible.	(**O**) Some men are not infallible.
(**O**) Some statements are not true.	(**I**) Some statements are untrue.

Notice that **A** changes into **E** ; **E** into **A** ; **I** into **O** ; and **O** into **I**. Obversion changes the quality of a proposition without changing its meaning.

Conversion. In this form of inference the predicate becomes subject and the subject becomes predicate. **Rule :** The quality of the proposition must remain unchanged, and no term must be distributed in the converse which was not distributed in the convertend. Let us remember that a negative distributes the predicate, but an affirmative does not. Hence **A** changes into **I** ; **E** changes into **E** ; **I** changes into **I** ; **O** cannot be converted, because by its conversion the subject (which is particular) becomes predicate of a negative proposition ; thus breaking the rule of the distribution of terms. **E** and **I** are said to be converted *simply* ; **A** is said to be converted *per accidens*, or by limitation, since the quantity is changed.

Conversion is legitimate on account of the Principle of Identity · for if two facts are united, it does not matter whether we say that the first is united to the second or the second to the first. The forms are as follows :

Convertend.	Converse.
(**A**) All S is P.	(**I**) Some P is S.
(**E**) No S is P.	(**E**) No P is S.
(**I**) Some S is P.	(**I**) Some P is S.
(**O**) Some S is not P.	Cannot be converted.
(**A**) All lawyers are graduates.	(**I**) Some graduates are lawyers.
(**E**) No vices are useful.	(**E**) No useful habits are vices.
(**I**) Some metals are liquids.	(**I**) Some liquids are metals.
(**O**) Some books are not interesting.	No direct conversion.

Notice that the proposition **O** may be converted indirectly by obversion, changing **O** into **I**. Thus : " Some books are not interesting " is equivalent to " Some books are uninteresting," and consequently, " Some uninteresting things are books."

Contraposition is a combination of obversion and conversion. By this process the contradictory of the original predicate becomes subject, and the original subject becomes predicate of the resulting proposition. **Rule** : Obvert the given proposition and then convert the obverse. As these processes are legitimate, the contrapositive must be true. Examples :

Given.	Obverse.	Partial Contrapositive.
(**A**) All S is P.	No S is not-P.	No not-P is S.
(**E**) No S is P.	All S is not-P.	Some not-P is S.
(**O**) Some S is not P.	Some S is not-P.	Some not-P is S.
(**I**) Some S is P.	Some S is not not-P.	None.

Notice that **A**, **E**, **O** have a contrapositive, but **I** has not, because by obversion it becomes an **O** proposition.

Again, by obverting a partial contrapositive we obtain a full contrapositive. Thus in the example above, " no not-P is S " obverted becomes " all not-P is not-S ; and the same with the rest. Hence we see that contraposition is a means for reversing the positive, or negative, or both terms of a proposition without changing its meaning. Notice that a proposition of the type we called " U " in Hamilton's system admits of reverting both terms, positive or negative, without conversion ; for as Stock says, " when two terms are co-extensive, whatever is excluded from the one is excluded also from the other. For instance : all equilateral triangles are equiangular, therefore all non-equilateral triangles are non-equiangular." [1]

In addition to contraposition we should mention here the obverted converse. This combination of conversion and obversion is peculiarly suitable to transform **I** and **E** propositions.

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[1] St. George Stock, *Logic*, p. 224.

Inversion means inferring from a given proposition, another whose subject is the contradictory of the subject of the original proposition. The given proposition is called invertend, the inferred proposition is called inverse.

The rule of inversion is : Convert either the obverted converse, or the obverted contrapositive.

We have no direct means of arriving at an equivalent proposition whose subject-term is the contradictory of the subject-term of the given proposition. We must therefore proceed indirectly, through the forms of eduction already considered. That is, the **S** must become predicate by conversion ; then not-**S** by obversion; finally not-**S** becomes subject by conversion again. This is what we mean by " Convert the obverted converse." This process is used in the inversion of the proposition **E**. In the proposition **A** we must use the other process, namely " Convert the obverted contrapositive." [Notice that **I** and **O** cannot be inverted.] Examples :

Given : (**E**) No plants are sensitive.
Converse : No sensitive things are plants.
Obverse : All sensitive things are not-plants.
Converse : Some not-plants are sensitive (which is the inverse of the *given proposition*).
Given : (**A**) All flowers are attractive.
Obverse : No flowers are unattractive.
Partial Contrapositive : No unattractive things are flowers.
Obverse : All unattractive things are not-flowers.
Converse : Some not-flowers are unattractive, or Some not-flowers are not attractive (which is the inverse of the *given proposition*).

Observe that hypothetical propositions are capable of transformation by the various kinds of Eduction. For example :

Given : (A) If a man is scientific, he must be logical.
Obverse : If a man is scientific, he cannot be illogical.
Partial Contrapositive : If a man is illogical, he cannot be scientific.
Obverse : If a man is illogical, he must be unscientific.
Inverse : If a man is unscientific, he may be illogical.
Obverse : If a man is unscientific, he may not be logical.

It is a matter of indifference whether we place the quantity with the consequent or with the antecedent.

13. **Material Immediate Inference.** The various kinds of inference so far considered are formal, that is, independent of the matter of the proposition, and can be worked out by mere symbols. They are based on the Laws of Contradiction, Excluded Middle and Identity, applied to combinations of the quantity and quality of propositions, the transposition of terms, and the changing of terms into their contradictories.

However, by changing the terms or employing contraries instead of contradictories, we can make other kinds of inference which are certainly correct, but whose consistency will cease to be formal, and will depend entirely on material considerations. To test those inferences we require a full knowledge of the matter, which in most cases is derived from experience or induction.

There are many such ways of passing from one proposition to another :

(a) **By a change of expression.** Thus : " I signed a document," or " The document was signed by me." Changes like this may be called grammatical conversions, and include a kind of inference when done intelligently.

(b) **By Converse Relation,** that is, turning a real relation (physical or moral) the other way about. Thus " Socrates was the teacher of Plato, and Plato was the pupil of Socrates. Bombay is North of Goa, and Goa is South of Bombay."

(c) **Inference by Modality** is based on the connection that exists between a fact and possibility, or between necessity and fact. But there is no inference from possibility to fact, nor from fact to necessity. For instance : Natural selection is a hypothesis or theory, possibly true, which can be used to explain variations in nature ; but because we can explain things in this way, we cannot infer " therefore a law of natural selection exists," for this would be passing from possibility to fact. But to say ; " This picture is a perfect piece of art ; therefore the painter of it is an artist," is a correct inference from fact to possibility, for the work reveals his power. Similarly it is right to infer that : " A stone left to itself is bound to fall, therefore it will fall." But it is wrong to argue : " He joins the army, therefore he does it of necessity."

(d) **Inference by Contraries** consists in drawing a contrary inference about a contrary subject. For instance : " Heat expands bodies, therefore cold contracts them ; Prudent people are pleasant, hence imprudent people are unpleasant." In each case there is no formal inference ; for the principle of contradiction does not apply to contrary terms. As to material inference, the propositions : " Heat expands bodies ; Cold contracts them " are consistent with facts. The inference, however, as stated is not certain, till we ascertain the law of contraction by experience. The inference about imprudent people is a senseless one, not even borne out by facts.

(e) **Inference by Added Determinants** means inferring a new proposition with an adjective or determining clause applied to both terms of the given proposition. Examples : " A man is a fellow creature, hence an old man is an old fellow creature ; A magistrate is a man, therefore a good magistrate is a good man." It is plain that the inference rests on material considerations. The rule is that the added determinants should be applied exactly in the same sense in both terms, in order to be correct. Thus the first example is correct ; the latter is wrong, since the adjective " good " as applied to the magistrate means good in his profession ; as applied to a man it means good in conduct ; consequently the inference fails.

(f) **Inference by Complex Conception,** either in subject or predicate or in both, as when I say : " Lawyers are graduates, hence the majority of lawyers are the majority of graduates. (Nonsense.) Again : " Homer was blind and the greatest of poets ; therefore, Homer was the greatest of blind poets." (Correct.) The guiding rule is the same as before, namely, that the complex concept should not change the meaning.

(g) **From a Distributive to a Collective sense and vice versa,** we may conclude wrongly or rightly. Thus it is right to say : " Each thing depends on God, therefore the whole collection of things depends on Him ", for such dependence is absolute and undisputable. But it is wrong to say : " A single document does not produce certainty, therefore neither does the whole collection " ;

for each document adds moral strength to the evidence of the others.

Again, from **Collective** to **Distributive**, it is wrong to say: "The assembly has the power to enact laws, therefore each member likewise has", for the power resides in the body.

(h) **Inference by Connotative Subalternation.** Connotative subalternation means the subalternation that exists between genus and species; that is, the connotation of the genus is included in that of the species. Hence what is true of the genus is true of the species, e.g., Man is an animal; therefore he is living, he is a substance. Again what is false of the genus is false of the species. Thus, since no vice can be rightly commanded (genus) it follows that a particular vice, such as stealing (species), cannot be rightly commanded.

CHAPTER VII

THE SYLLOGISM

1. What Reasoning is. Inference, reasoning and proof are different names for the same thing, namely, the process of arriving at some truth (conclusion) by means of other truths already known (premises). Looking at this process from the conclusion up to the premises, it is called *proof*; the same process viewed from the premises down to the conclusion is *reasoning*; and the connection that exists between premises and conclusion is *inference.*

Observe that in this act, characteristic of the human mind, the three mental operations are subordinate. The simple concepts are first required; these are united into a proposition, by agreement or disagreement; this proposition then leads to another proposition derived immediately from the very relation which the two terms of the given proposition hold to each other, as explained in the previous chapter.

But the intellect goes further; it sees the connection between two propositions and naturally draws the conclusion implied by them. Or again, penetrating into the nature of facts, it grasps a hidden relation and makes a general statement. Hence there are two kinds of reasoning which proceed in two opposite directions:

(1) We can begin with a number of **Particular** truths, and work up from them to a **General** truth; and this

is called **Induction.** Thus Galileo, by repeated experiments with falling bodies, concluded that "Gravity imparts the same velocity to **all** falling bodies."

(2) We can begin with a **General** truth and work down from it to a **Particular** truth; and this is called **Deduction.** Thus from Galileo's proposition (just mentioned) about **all** falling bodies, we can draw conclusions about particular bodies.

Although Induction is first in the order of nature, it depends on material experiences, and does not readily lend itself to formal treatment. For this reason we relegate Induction to a later place in our treatise, and confine ourselves for the present to Deduction.

In a previous section we have spoken of **immediate inference,** which can be drawn from a single proposition, and therefore does not need a syllogism. Syllogistic reasoning, on the contrary, is **mediate inference,** because it is drawn from a first proposition through the medium of a second.

2. **Mediate Inference** is, therefore, the process of drawing a conclusion from two (or more) propositions, which are the ground of the conclusion. That this process is mediate and not immediate appears from the fact that it requires three terms in the premises: a subject, a predicate, and a third term to serve as a bridge between the premises and the conclusion. Let me make this clear by examples:

1. A question existed until recent times whether swamps were malarious. The relation between subject and predicate became known by two propositions with a common term. On investigation it was found that swamps were breeding a certain kind of mosquito, and furthermore that those

mosquitoes by their bite introduced certain parasites into the blood, thus causing malaria. The three propositions are connected as follows :

> A certain kind of mosquito is a source of malaria.
> Swamps breed that kind of mosquito.
> Swamps are a source of malaria.

2. The notion of democratic government may occur to us as being popular. The proposition : " Democracy is popular", will become clear by means of a third notion showing the connection between subject and predicate. Examining the subject, we see that democracy defends liberty, and we know otherwise that the sense of liberty appeals strongly to human nature. Hence the argument will stand thus :

> What promotes liberty is popular,
> Democracy promotes liberty,
> Democracy is popular.

3. Again, the statement : " No man is a good judge of himself", is placed in a clear light by showing that every man is prejudiced towards himself, and at the same time observing that a prejudiced person cannot be a good judge. The whole process runs this way :

> Whoever is prejudiced is not a good judge,
> Every man is prejudiced towards himself,
> No man is a good judge to himself.

4. It is a moot-question whether capital punishment ought or ought not to be abolished. But it is clear that a measure necessary to preserve society should on no account be abolished ; and it can reasonably be urged that the present state of society requires capital punishment to check crime. Hence the argument :

> A necessary measure should not be abolished,
> Capital punishment is a necessary measure,
> Capital punishment should not be abolished.

Observe that by the medium of a third term we can arrive not only at affirmative conclusions but also at negative ones.

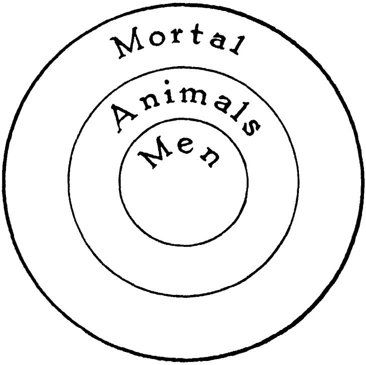
3. **Definition of the Syllogism.** A syllogism is a form of argument consisting of three propositions, so related to one another that two of them being laid down, the third necessarily follows from them. Or, a form of reasoning in which two terms are compared with a third term, and their mutual agreement or difference is deduced therefrom. Examples :

All animals are mortal,	M is P,
All men are animals,	S is M,
All men are mortal.	S is P.

The Terms and Propositions of the Syllogism. (1) The terms are three, called major, minor and middle. The major is the predicate of the conclusion, the minor the subject of the conclusion ; the middle is the one repeated in the premises and not found in the conclusion. (2) The propositions are also three ; the major premise, the minor premise and the conclusion. The major proposition contains the major term and the minor contains the minor term.

4. **Internal Form of the Syllogism.** Looking more closely into syllogistic reasoning, we notice that the inference consists in a real connection among the propositions by means of the middle term. What happens is that the middle term expresses the cause, ground or reason why the major term is united to the minor in the conclusion. Thus, the reason why Socrates is mortal is discovered in the fact that he is an animal ; modesty is praiseworthy, because it is a virtue ; education is beneficial to society inasmuch as it develops man's faculties.

This connection may be shown to the eye by a graphic representation. Let us take the previous syllogism. The major term "Mortal" may be represented in its full extension by a large circle; the middle term "Animals" by a smaller one included in the first, according to the meaning of the first premise: "All animals are mortal." Again the minor term "Men" is included in the circle "Animals," as stated in the second premise; "All men are animals." Hence it is plain that "men" is included in "mortal", which is the conclusion. It is also seen in the figure that "animals" stands in the middle between the two extremes. Such is the mechanism of the syllogism.



5. Maxims concerning Inference. Observe carefully some important maxims that follow from the above explanation of syllogistic reasoning and hold good as general principles for any inference whatever:

First, that one of the premises contains the conclusion implicitly, while the other premise makes it explicit. By saying, for instance, "No men are infallible", it is stated implicitly that "doctors are not infallible"—which becomes explicit by declaring that "doctors are men."

Secondly, the conclusion can never surpass the certainty of the premises from which it is derived, and by which it is supported.

Thirdly, the mind assents to the truth of the conclusion on the evidence of the premises ; hence it is that those premises which make known the conclusion, must themselves be clearer to the mind than the conclusion. And here is the reason why the first principles of science, being self-evident, cannot be demonstrated, since their truth cannot be derived from another clearer truth. Such is the principle, for instance, " The whole is greater than its part ", and all plain facts of experience as " Iron is heavier than water ", " Men are bigger than mice ", etc., which neither can, nor need be proved.

Fourthly, sound reasoning essentially requires that both premises be true, for they are the cause and the ground on which the conclusion is what it is. People, it is true, do sometimes argue from false premises and yet reach a conclusion which is true to fact, and that, moreover, by a correct use of the syllogistic form. In such a case the truth of the conclusion is an accidental coincidence with facts. It is not really grounded on the premises, because out of falsity truth cannot emerge.

This point is of great importance, for it shows that a mere correct use of syllogistic reasoning does not necessarily result in truth. We must look not only to the correctness of the process of inference, but also to the truth of *both* the premises.

Hence the two rules :

(1) When the premises are both true, a false conclusion cannot logically be drawn from them.

(2) If one or both premises are false, the conclusion may be true *per accidens* ; but it will never be

true *per se*, even if the inference has been drawn according to the formal rules of the syllogism.

In short : From true, only true ; from false, either true or false may follow. Examples :

All the nations of Europe engaged in the world-war, (false)
The United States of America is a nation of Europe, (false)
The United States of America engaged in the world-war. (true)

Agriculture is an exact science, (false)
Mathematics is Agriculture, (false)
Mathematics is an exact science. (true)

No inhabitants of Bombay are Mahommedans, (false)
The Governor is an inhabitant of Bombay, (true)
The Governor is not a Mahommedan. (true)

Hence it follows that even if the conclusion is true, the arguments or premises are not necessarily true.

6. Fundamental Principles of the Syllogism. The syllogism, as defined above, is based on the following principle : *Dictum de omni, dictum de nullo.* Whatever is affirmed or denied of a universal subject may be affirmed or denied of each of the particulars or classes contained under that subject. This principle shows the connection of the three propositions, and how we descend from a general statement to a particular one.

We may view the syllogism in another aspect ; viz. as being a comparison of terms. Then the principle is expressed as follows : Two things which are identical with a third thing are identical with each other ; and : Two things, one identical with, and the other different from a third thing, are different from each other. The first expression refers to an affirmative conclusion, the second to a negative conclusion.

The reasoning is reduced to a formula of substitution like this:

A is B, and B is C; therefore A is C.
A is B, and B is not C; therefore A is not C.

Notice that the principles underlying the legitimacy of the syllogism, whether regarding propositions or terms, are supported by the principle of contradiction, as can easily be shown.

7. Rules of the Syllogism.

Concerning structure. (1) There must be three terms only. Proof: A syllogism is a comparison of two terms with the middle term. This comparison requires three terms only.

(2) There must be three propositions only. Proof: The definition of the syllogism includes only three propositions.

Concerning quantity. (3) No term must have greater extension in the conclusion than it has in the premises. Proof: Any greater extension in the conclusion is not guaranteed by the premises, since it does not correspond to the middle term. Another reason: A syllogism thus formed will contain more than three terms.

(4) The middle term must be distributed at least once in the premises. Proof: Otherwise the two uses of the middle term may be different; and if so, the comparison will be made with no common middle.

Concerning quality. (5) From two negative premises no conclusion follows. Proof: The extremes differ from the middle. Hence nothing is known as to their mutual agreement or disagreement, so no conclusion is possible.

(6) If one premise is negative the conclusion must be negative ; and if the conclusion is negative one premise must be negative. Proof : In the first case one term agrees and the other disagrees with the middle ; hence the extremes disagree, or the conclusion is negative. In the second case we find in the conclusion that the extremes disagree ; hence one of them must disagree with the middle, or one premise must be negative.

Corollaries. (7) No conclusion can be drawn from two particular premises. Proof : If both are negative, no conclusion follows (Rule 5). If both are affirmative, the middle term is not distributed ; hence there is no conclusion (Rule 4). If one is affirmative and the other negative, then there is only one term distributed, which must be the middle ; hence no distributed term is left for the conclusion. But the conclusion must be negative (Rule 6) ; therefore no conclusion is possible.

(8) If one premise be particular, the conclusion must be particular. Proof : Three cases are possible ; premises **AI, EI, AO**. In the first case the only distributed term goes to the middle. In the second and third, the two distributed terms go to the middle and to the predicate of the conclusion, which must be negative. Hence the subject of the conclusion must be particular in all these three cases.

8. Distinction between Figure and Mood. Figures are forms of the syllogism determined by the position of the middle term. Hence only four figures are possible.

Moods are forms of the syllogism determined by the quality and quantity of the three constituent propositions. The valid moods are four in the first figure, four in the second, six in the third, and five in the fourth :

First Figure, in which the middle term is subject in the major premise and predicate in the minor.

EXAMPLE :

M—P, All men are fallible,
S—M, All doctors are men,
S—P. All doctors are fallible.

Particular Rules : (1) The major premise must be universal. Reason : to have a distributed middle. (2) The minor premise must be affirmative—otherwise the major term becomes illicit in the conclusion.

Moods : **A A A,** **E A E,** **A I I,** **E I O.**
(Barbara) (Celarent) (Darii) (Ferio)

Second Figure, in which the middle term is twice predicate.

EXAMPLE :

P—M, No vices are commendable,
S—M, All virtues are commendable,
S—P. No virtues are vices.

Particular Rules : (1) One premise must be negative. Reason : in order to have a distributed middle. (2) The major premise must be universal—because the conclusion is negative, whose predicate is the subject of the major premise.

Moods : **E A E.** **A E E,** **E I O,** **A O O.**
(Cesare) (Camestres) (Festino) (Baroko)

Third Figure, in which the middle term is twice subject.

EXAMPLE :

M—P, All men are rational,
M—S, All men are fallible,
S—P. Some fallible beings are rational.

Particular Rules : (1) Minor premise must be affirmative—because if it were negative, the conclusion would be negative, with illicit process of major term. (2) The conclusion must be particular—because the minor premise is affirmative, whose predicate is the subject of the conclusion.

Moods :	**A A I,**	**A I I,**	**I A I,**	**E A O,**
	(Darapti)	(Datisi)	(Disamis)	(Felapton)
		O A O,	**E I O.**	
		(Bokardo)	(Ferison)	

Fourth Figure, in which the middle term is predicate in the major premise and subject in the minor.

EXAMPLES :

P—M, 1. All lawyers are graduates,
M—S, All graduates are educated,
S—P. Some educated persons are lawyers.

2. No lawyers are undergraduates,
Some undergraduates are educated,
Some educated persons are not lawyers.

Particular Rules : (1) If the major premise is affirmative the minor must be universal. Reason : in order to have a distributed middle. (2) If the minor premise is affirmative the conclusion must be particular. Reason : in order to avoid illicit process of the minor term. (3) In negative moods the major premise must be universal. Reason : in order to avoid illicit process of the major term.

Moods : A A I, (Bramantip) **A E E,** (Camenes) **I A I,** (Dimaris) **E A O,** (Fesapo)
E I O.[1] (Fresison)

An examination of all the figures and moods shows :

(1) An **A** proposition can be proved only in Figure **I**, and only by one mood.

(2) An **E** proposition can be proved in three figures and by four moods.

(3) An **I** proposition can be proved in three figures and by six moods.

(4) An **O** proposition can be proved in all four figures and by eight moods.

Again, since **A** and **O** are contradictory, it is correct to say that **A** is difficult to prove and easy to disprove, while **O** is easy to prove and difficult to disprove.

Notice also that an **E** proposition is highly efficient as a major premise, as it holds that place in eight moods, two in every figure. Its power of combination is exceptional owing to both its terms being distributed.

9. Characteristics of the Figures. The first figure is the most natural and valuable of all the figures, because : (1) It is the only figure to which the double basis of the syllogism, *Dictum de omni, de nullo* applies. (2) It is the only figure that proves all and any of the four propositions ; and above all, it is the only figure that proves a general connection between subject and predicate, which is the constant effort of thought and research. (3) In this figure the subject of the conclusion is subject in the premises, and the predicate of the conclusion is also predicate in the premises, and the middle term is really middle between the major and minor terms. These reasons explain the fact that arguing in this figure is more natural than in any other. It was for all these reasons, but especially for the first, that Aristotle recognised the first figure as the only strictly valid one. The second and third he admitted as valid only so far as they could be logically reduced to the first.

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[1] The words " Barbara, Celarent," etc., are mnemonic names for the moods. In these traditional names the *three vowels* denote the three propositions of which the mood is composed. The *consonants* refer to the process of Reduction, of which hereafter. These names became current in the schools as far back as the thirteenth century, and were arranged in four Latin hexameters, one for each figure, to help the memory.

The second figure proves only negative conclusions ; but the position of the middle term, appearing twice as predicate, shows very naturally the opposition between the major and minor terms, since it states that one agrees and the other disagrees with the middle. Hence it proves distinctions between things.

The third figure proves only particulars. It proves instances under the general rule, or exceptions to the rule. This figure is natural only when the middle term is singular or definite in quantity.

The fourth figure has no special characteristics, except that it is most imperfect, and is actually rejected by many logicians.

10. Possible Moods. The problem is to find out all the possible combinations with the four propositions **A**, **E**, **I**, **O**, in groups of three. Beginning with the premises, we have 4 × 4 = 16 possible pairs. Then 16 × 4, on account of four possible conclusions, make 64 different moods. Extending the idea of mood to the varieties in the four figures, we reach 64 × 4 = 256 possible moods.

11. Valid Moods. One method to determine which moods are valid is to test each of the possible moods by the rules of the syllogism.

An easier and shorter method is as follows : Combine the four propositions into 16 pairs :

Major	A	A	A	A	E	E	E	E	I	I	I	I	O	O	O	O
Minor	A	E	I	O	A	E	I	O	A	E	I	O	A	E	I	O

and draw from each a valid conclusion according to the general rules of the syllogism. A certain number will drop out, as being two particular or two negative premises. Moreover the pair **I E** has no conclusion, on account of the illicit process of the major term. Hence only eight moods are left. But the pairs **A A, E A** in Figure III may yield a particular conclusion which is not weakened ; consequently the valid moods are ten. Again, some of these moods are valid in various figures. Considering these varieties as new moods, we obtain nine more, which together with the previous ten make in all nineteen valid moods. Notice that the mood **A E O** has been passed over, because its particular conclusion becomes weakened in every figure.

12. Strengthened and Weakened Moods are alike, so far as both yield a particular conclusion and a valid one from two universal premises ; but there is a considerable difference between the two :

A strengthened mood concludes particularly, because it cannot do otherwise ; while a weakened mood draws a particular conclusion instead of a universal one that could legitimately be drawn. Thus in the former there is an excess of strength in the premises ; in the latter, weakness in the conclusion.

The weakened moods are also called subaltern, since the particular conclusion is subaltern of the universal as in **A A I**, **A A A** (Figure I). Weakened moods, though valid, are generally rejected from the nineteen, on the ground of being useless and misleading. There are two in Figure I, (Barbara, Celarent) ; two in Figure II, (Cesare, Camestres) ; and one in Figure IV, (Camenes), which may become weakened moods.

The strengthened moods, on the contrary, are admitted into the nineteen, because they afford a variety of reasoning. Yet they are unnecessary, for each may be substituted by another in the same figure to prove the same conclusion. Thus in Figure III Darapti may be replaced by Datisi or Disamis ; and in Figure IV Fesapo can be replaced by Ferison.

Summing up, we find that fifteen moods out of the nineteen are fundamental—in which the middle term is once distributed, and both extremes are of equal extension in premises and conclusion. Four moods are strengthened : Darapti, Felapton (Figure III), and Fesapo (Figure IV), in which the middle term is twice distributed ; and Bramantip (Figure IV) in which the major term is distributed in the premises and not distributed in the conclusion.

13. Reduction of Syllogisms. Reduction is a process by which a given syllogism in any of the imperfect figures is changed into some mood of Figure I. *Direct* or *ostensive* reduction is accomplished by means of transposition of premises and by conversion, both being legitimate manipulations. How this must be done in each particular case is suggested in the mnemonic names as follows : The first consonant means that the given mood will be changed into the mood in Figure I that begins with the same consonant, i.e., Cesare and Camestres to Celarent ; Festino to Ferio, etc. The letter **m** means transposition of premises ; **s** means simple conversion ; and **p** means conversion *per accidens* of the precedent vowel or proposition. Two moods only do not come under this rule, viz., Baroko and Bokardo, on account of the **O** proposition. Examples :

1. Cesare.	2. Camestres.
No superstition is science.	Astronomy is a science,
Astronomy is a science,	No superstition is science,
Astronomy is not superstition.	No superstition is astronomy.
Reduction to Celarent.	**Reduction to Celarent.**
No science is superstition,	No science is superstition,
Astronomy is a science,	Astronomy is a science,
Astronomy is not superstition.	Astronomy is not superstition.

Baroko and **Bokardo** may be reduced directly by following the indication of the mnemonic names **Faksoko** and **Doksamosk,** where the letter **k** stands for obversion. Examples :

Faksoko (reduced to) Ferio.

All virtuous people are happy,	No unhappy people are virtuous,
Some rich people are not happy,	Some rich people are unhappy,
Some rich people are not virtuous.	Some rich people are not virtuous.

Doksamosk (reduced to) Darii.

Some cyclones are not foreseen,	All cyclones are dangerous,
All cyclones are dangerous,	Some unforeseen phenomena, are cyclones,
Some dangerous phenomena are not foreseen.	Some unforeseen phenomena are dangerous.

The conclusion : " Some unforeseen phenomena are dangerous " becomes (by conversion) : " Some dangerous phenomena are unforeseen " ; and this (by obversion) : " Some dangerous phenomena are not foreseen," which is the conclusion of the first syllogism. A clumsy reduction, but it shows the unity of the process adapted by Aristotle.

14. Indirect Reduction. It is also called *per impossibile*, or *ad absurdum*, and consists in showing that the conclusion must be accepted as true, because in the supposition of it being false an absurdity or contradiction follows : Example in Baroko :

(i) All graduates are educated,
(ii) Some workmen are not educated,
(iii) Some workmen are not graduates.

If the premises are true, the conclusion must be accepted as true. For if (iii) is false, its contradictory must be true, viz. : (iv) All workmen are graduates. Joining this proposition with (i) we have the following syllogism :

(i) All graduates are educated,
(iv) All workmen are graduates,
(v) All workmen are educated.

This syllogism, being in Barbara, is formally correct. But proposition (v) is the contradictory of (ii), and two contradictories cannot be true together. Proposition

(ii) was given as true, hence proposition (v) must be false—not for any fault in inference, but because of the premises. Now premise (i) is true ; therefore premise (iv) is false, and its contradictory (iii) must be true.

Observe that indirect reduction may be applied to any mood of the 2nd, 3rd and 4th figures, and the result will be a mood in the first figure.

15. The Use of Reduction. The moods of the first figure are acknowledged to be perfect, while all others are imperfect, as Aristotle himself says. For in the first place, a syllogism in the first figure shows clearly and directly the inference of the conclusion from the premises ; and (2) the major contains the conclusion ; this is evolved through the minor, and assumes the quality of the former and the quantity of the latter. The inference in the other figures is certainly legitimate but not so clear, and is not demonstrated by the *Dictum de omni*. Their legitimacy is proved to demonstration by means of reduction. According to Aristotle, therefore, and for any one who puts aside all other canons but those of the first figure, reduction would be required as a conclusive proof of the validity of the imperfect figures. The "dictum," however, is not the only general principle underlying correct inference in all syllogistic reasoning ; and consequently reduction is not absolutely necessary.

Nevertheless the methods of direct and indirect reduction are useful in many ways. In particular, they have the merit of exhibiting the unity that exists among the various combinations of our thoughts—since all can be reduced to one fundamental form of reasoning.

CHAPTER VIII

ANALYSIS OF ARGUMENTS

1. Difficulties in the Way. We have so far analysed the syllogism in its most simple form, with a view to explain better the nature of our reasoning, the general laws of correct inference and their bearings on the various syllogistic moods.

Let us begin by pointing out some of the difficulties which stand in the way of logical analysis. The syllogism in its logical form is a rather theoretical and ideal expression of thought. Speakers and audiences, writers and readers are naturally disinclined to it. Furthermore, the mind hardly ever pays attention to distinctions of figures and moods in the act of working out its arguments. This is done only by reflection.

Language, moreover, prompts us to express thought and feeling in a great variety of styles, and to employ rhetorical images, maxims, analogies and examples to enforce the point at issue.

This does not mean that Dialectic and Rhetoric are opposed; they really combine together, lending each other support. The two Arts, no doubt, have different ends in view, viz., the convincing of the mind and the persuading of the will respectively. Dialectic looks at valid inference, while Rhetoric works on emotions and feelings. Both go naturally hand in hand, though now one, now the other may be more conspicuous in the speech.

Hence it comes about that the processes of reasoning, in the course of ordinary life and conversation, are often disguised under the free forms of rhetorical or oratorial speech. Consequently it becomes necessary for us to practise the art of picking out the lines of reasoning, and reducing them to the scientific forms of the syllogism, in order to test their validity and detect any fallacies which may lurk under a plausible surface-presentment.

2. Kinds of Argument. A large number of reasonings are composed of complex propositions in which the terms of the conclusion are not the same as those of the premises; and yet the argument itself may be valid and legitimate.

Very often too our reasonings are abbreviated by suppressing one or more propositions which form part of the meaning of the speech. At times several inferences are combined to make a compound argument with a simple conclusion. In both cases the exercise of the logician will consist in supplying the missing links so as to develop the argument or arguments into a complete form.

The categorical proposition is frequently joined to other kinds of propositions, thus forming mixed syllogisms, whose inference is tested by definite rules to be proposed later on.

In the last place comes the question of probable reasoning, extensively used by speakers and writers; and sometimes abused by them, whenever an inference is grounded on some sort of connection which may or may not be real, and which the mind can only estimate roughly while listening.

3. The Testing of Inference. Testing an inference means finding out whether the conclusion follows from the premises. Hence we must first ascertain what is laid down as a conclusion, and what are the reasons given as premises. If the propositions are complex, it will be helpful to reduce them to a simple form with clean-cut terms, dropping out those parts which do not belong to the inference, or adding that which is understood. The logician must be exact in adding what the author is understood to mean, and in rendering the thoughts in other words. Once this is done, we shall be able to throw the argument into any of the recognised forms, and apply to them the rules of Logic. This is all that can be said in general. The process, in particular cases, must be worked out each one by itself. Examples :

1. The divine law commands us to honour our rulers,
George V is our ruler ;
The divine law commands us to honour George V.

Making " rulers " the subject in the major premise, the argument runs as follows :

Our rulers are to be honoured by divine law,
George V is our ruler ;
George V is to be honoured by divine law.

2. A man is not a law to himself ; for a law requires superior and inferior, and a man is neither superior nor inferior to himself.

The argument is shown to be correct, by giving the definition of a law implied in it :

A law is a command of a superior to an inferior ; the command of a man to himself is not a command of a superior to an inferior ; therefore the command of a man to himself is not a law.

3. Who can say that suffering is always evil, if he admits that remorse involves pain, and yet may sometimes be a

real good ? The meaning is that suffering is not always an evil ; and this is the conclusion of the two admitted propositions, that remorse is painful and that remorse is sometimes a real good. The argument is :

Some remorse is not evil (is really good),
Remorse is suffering (painful) ; therefore
Some suffering is not evil (suffering is not always evil).

4. The energy of matter in the plant produces the vital actions of the plant ; but the same matter (i.e., the same chemical elements that are in the plant), is found outside the plant in the inorganic world ; therefore, the energy of matter alone, as found in the inorganic world, produces the vital actions of the plant ; consequently there is no need of any principle of life distinct from matter. (Haeckel).

The conclusion with the particle " alone " is illicit, as not at all grounded on the major premise. It is the energy of matter combined with life which produces vital action.

4. The Enthymeme. Common language naturally avoids the logical form, as we have seen ; but it goes further. Premises are very often omitted in the course of reasoning. It is left for the logician to supply the missing premises in order to test the inference. The most frequent form is the enthymeme, which may be defined : " An abridged syllogism in which one of the three propositions is not stated, because it is understood." The suppressed proposition may be the major, the minor or the conclusion ; and accordingly the enthymeme is styled of the 1st, 2nd, or 3rd order. Examples :

1st Order. Some criminals are not punished, because they are unknown.

2nd Order. Some criminals are not punished, because unknown persons cannot be punished.

3rd Order. Unknown persons cannot be punished, and some criminals are unknown.

The syllogism for the three cases is :

No unknown persons are punished,
Some criminals are unknown,
Some criminals are not punished.

Observe that for practical purposes all the three forms are common, but in most cases a statement is laid down with the reason for it [as in Nos. 1 and 2]. That reason contains the middle term and one extreme, while the statement is the conclusion ; hence you can easily find out the missing proposition. When the two premises are given, the conclusion naturally follows.

Take as an illustration the following examples :

1. Save thee I could. Could I not then destroy ? The meaning is, I could save you, therefore I could destroy you ; an enthymeme of the first order.—The full syllogism is as follows : All who are able to save are able to destroy ; but I am one who was able to save you, therefore I am one who was able to destroy you.

2. The universe is the work of a rational agent ; for no irrational agent could have produced a work which manifests design ; an enthymeme of the second order.—The argument fully re-stated is as follows : All things which manifest design are the work of a rational agent ; but the universe is a thing which manifests design ; therefore the universe is the work of a rational agent.

3. Slavery is immoral ; and to retain a servant against his will is a kind of slavery ; an enthymeme of the third order.—By adding the conclusion we have : All kinds of slavery are immoral : but to retain a servant against his will is a kind of slavery ; therefore to retain a servant against his will is immoral.

An enthymeme may even consist of one single proposition, or enthymematic expression where emphasis is laid on a special noun or adjective, thereby suggesting a reason. Examples :

1. O *mortal*, cherish not immortal hate.—You cannot cherish immortal hate ; because you are mortal.
2. Is it not love *to do so much* for you ?—I love you ; since I have done much for you.
3. No one believes that *ridiculous* thing.—No one believes that thing ; because it is ridiculous.
4. Have thou nothing to do with that *just* man.—Have nothing to do with that man ; for he is just.
5. If there is no *sorrow*, there is no amendment.—There is no amendment ; because there is no sorrow.

5. **The Epichireme.** This is a syllogism in which a reason is given for the truth of either one or both of the premises ; in other words, one or both premises are enthymemes. Each enthymematic premise is tested by the rules of the syllogism, and so also is the fundamental syllogism. Examples :

1. The whole of Cicero's speech " pro Milone " is reduced to an argument of this kind : It is lawful to slay one who lies in wait for us, for it is according to natural law and to the law of nations. But Claudius had lain in wait for Milo, as is shown by all the circumstances ; therefore it was lawful for Milo to slay him.
2. All books are liable to error, because they are human productions.
   And books are useful, for they impart knowledge.
   Hence, there are useful things that are liable to error.

This form of argument is useful to a speaker who advances doubtful propositions, or difficult ones. For by pointing out the reasons, he restrains the impatience of the hearers, and makes them ready to wait for the proofs to be developed.

6. The Polysyllogism. The name itself contains its own definition, viz., a plurality of syllogisms, or a chain of reasoning in which the conclusion of one syllogism is used as a premise in the next. The preceding one is called Prosyllogism ; the following, Episyllogism. Example :

All moral creatures are to be treated with respect,
Rational beings are moral beings ;
Rational beings are to be treated with respect,
Servants are rational beings ;
Servants are to be treated with respect.

It is plain that a Polysyllogism can be abridged in the form of an Epichireme. Thus we can say :

Rational beings are to be treated with respect ; for they are moral ;
Servants are rational beings ;
Servants are to be treated with respect.

7. The Sorites. Sorites (heap) is a chain of arguments in which some premises and all intermediate conclusions are omitted till the last proposition, which is the conclusion of the whole argument. There are two forms :

	ARISTOTELIAN.	GOCLENIAN.	
Minor.	Socrates is a man,	A moral being is responsible,	*Major.*
Major.	A man is a rational animal,	A rational animal is moral,	*Minor.*
Major.	A rational animal is moral,	A man is a rational animal,	*Minor.*
Major.	A moral being is responsible,	Socrates is a man,	*Minor.*
Concl.	Socrates is responsible.	Socrates is responsible.	*Concl.*

In the Aristotelian method the predicate of each premise becomes subject of the next, and the subject of the first is subject of the conclusion, while the predicate of the last is predicate of the conclusion. The argument, although apparently in the Figure IV, is really in Figure I, as is manifested by the conclusion.

In the Goclenian method the premises run in inverse order. In reality both are the same argument, and may be developed into as many syllogisms as propositions, less two.

A man is a rational animal,
Socrates is a man,
Socrates is a rational animal.

A rational animal is moral,
Socrates is a rational animal,
Socrates is moral.

A moral being is responsible,
Socrates is a moral being,
Socrates is responsible.

A moral being is responsible,
A rational animal is moral,
A rational animal is responsible.

A rational animal is responsible,
A man is a rational animal,
A man is responsible.

A man is responsible,
Socrates is a man,
Socrates is responsible.

In the Aristotelian method the first premise is a minor as it contains the subject of the conclusion; and the intermediate propositions between the first and the last are major premises, as the middle term is subject in them. Each new conclusion, therefore, is used as a minor in the following syllogism.

In the Goclenian method the premises are placed in the reverse order. Consequently the first is a major; and the others are minor premises, as the middle term is predicate in them. Hence the new conclusions are used as major premises of the expanded syllogisms.[1]

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[1] The Aristotelian sorites is said to be *Progressive*; and by some logicians it is called *Inductive*. If it is developed into syllogisms the subject of each conclusion is the same. The Goclenian is said to be *Regressive* and *Deductive*. The conclusion of each syllogism has the same predicate. (See St. George Stock, *Logic*, pp. 352-60.)

Rule I. Only one premise can be particular, and must be that which contains the minor term. (The first in the Aristotelian, the last in the Goclenian.) Reason: (1) Only one: for if a second were used there would be two particulars. (2) It must be the minor premise; for if it were the major there would be undistributed middle.

Rule II. Only one premise can be negative; and it is that in which the major term occurs. (The last in the Aristotelian, the first in the Goclenian.) Reason: (1) Only one; for if a second were used there would be two negatives. (2) It must be the major premise, to prevent illicit process of the major term.[1]

The sorites is a beautiful argument, and useful to sum up the reasonings of a full dissertation; but it is also a means for leading almost insensibly to a wrong conclusion. In a heap of propositions it is quite easy to include a false one. Besides this, the inference may be vitiated through overlooking the strict logical rules.

From what is said above, it is plain that a sorites is an abridged polysyllogism. Example:

Whoever is prudent is temperate,
Whoever is temperate is constant,
Whoever is constant is imperturbable,
The imperturbable are free from sadness,
Whoever is free from sadness is happy; therefore,
The prudent are happy. (Seneca, ep. 85).

The same argument as a polysyllogism:

Whoever is temperate is constant,
Whoever is prudent is temperate; therefore,
Whoever is prudent is constant.

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[1] The rules given apply to a *Regular* sorites, which runs in Figure I. An *Irregular* sorites, which runs in other figures, is guided by other rules.

But whoever is constant is imperturbable ; therefore,
Whoever is prudent is imperturbable.
But the imperturbable are free from sadness ; therefore,
Whoever is prudent is free from sadness.
But whoever is free from sadness is happy ; therefore,
Whoever is prudent is happy.

8. Hypothetical Syllogism. We have considered so far only categorical syllogisms ; but reasoning is also conducted with hypothetical and disjunctive propositions. Hypothetical propositions, having quantity and quality, may be combined in figures and moods ; hence we may have pure hypothetical syllogisms in any figure, and apply reduction to them. The link with the middle term will always be a connection of dependence, not of identity in existence as in the categoricals ; yet the rules of the categorical syllogisms hold good for the hypothetical. Example :

If the civil laws are bad, the nation is not prosperous,
If the government is bad, the civil laws are bad : therefore,
If the government is bad, the nation is not prosperous.

A Mixed Hypothetical Syllogism is made up of an hypothetical major, a categorical minor and a categorical conclusion. Such syllogisms are quite common and have definite rules. (1) Affirming the antecedent in the minor, the conclusion affirms the consequent, not vice versa. (2) Denying the consequent in the minor, the conclusion denies the antecedent, not vice versa. To put it briefly : Affirm the antecedent or deny the consequent. Hence of the four possible moods only two are valid : one affirmative, the other negative. The old classical names are : *modus ponens, modus tollens*. One is constructive, the other is destructive.

The two rules of valid inference are made explicit thus :

(1) If I affirm the antecedent, I must affirm the consequent ; because the very nature of the hypothetical proposition tells me that the consequent necessarily follows from the antecedent ; therefore, granted the antecedent, the consequent must be granted. Not vice versa ; that is, if I affirm the consequent, I cannot affirm the antecedent ; for the consequent might follow from some other source. For instance : if a man is shot through the heart, he dies ; but he died ; therefore, he was shot through the heart. The conclusion does not follow, as he might die of any other cause.

(2) If I deny the consequent, I must deny the antecedent ; because the removal of the consequent implies that the antecedent was not present ; for, if the latter had been present, the former would also be. Not vice versa ; that is, if I deny the antecedent, I have no right to deny the consequent ; for, the consequent might follow from other antecedents. To go back to the previous example : If a man is shot through the heart, he dies ; but he was not shot through the heart ; therefore, he did not die. The conclusion is wrong, as he might die of some other cause.

These same rules may be tested by the general rules of the syllogism, by transforming the hypothetical into categorical. The affirmative mood becomes Barbara, and the negative one Camestres, while the pseudo-moods are invalid. Example :

If virtue is free, vice is free ; but virtue is free ; therefore vice is free.—Affirmative mood, *Modus ponens.*

Test for Rule I.

All who are free in virtue are free in vice,
John is free in virtue ;
John is free in vice (Fig. I, Barbara).

If virtue is free, vice is free ; but vice is not free ; there fore virtue is not free. Negative mood, *Modus tollens.*

Test of Rule II.

All who are free in virtue are free in vice,
An insane man is not free in vice ;
An insane man is not free in virtue (Fig. II, Camestres).

Pseudo-moods :

If virtue is free, vice is free ; but vice is free ; therefore virtue is free.—False affirmative mood. Proof :

All who are free in virtue are free in vice,
John is free in vice,
John is free in virtue (Fig. II, Undistributed middle).

If virtue is free, vice is free ; but virtue is not free ; therefore vice is not free. False negative mood. Proof :

All who are free in virtue are free in vice,
An insane man is not free in virtue,
An insane man is not free in vice (Fig. I, Illicit process of major term).

For the sake of exercise, notice that by replacing the major premise by its obverted contrapositive, the affirmative mood becomes negative in the same argument. Example :

If any man is wise he is honest ; but John is wise ; therefore he is honest. *Modus ponens.*

If any man is dishonest, he is unwise ; but John is wise ; therefore he is honest. *Modus tollens.*

The principle underlying the inference in such syllogism is called by some “ the law of reason and consequent.” This means that when two propositions are related as reason and consequent, the truth of the consequent follows from the truth of the antecedent, and the falsity of the antecedent follows from the falsity of the consequent. This view is practically safe, leaving aside the question whether the antecedent is the full reason for the consequent or not.

9. Disjunctive Syllogism. The pure disjunctive syllogism is of no use on account of the many limitations attached to the disjunctive proposition. But the mixed disjunctive is important. It consists of a major disjunctive and a categorical minor which either affirms or denies one of the alternatives ; the conclusion accordingly denies or asserts the truth of the other alternative.

In stating the rules of the disjunctive syllogism let us call to mind that a disjunctive proposition may be perfect or imperfect. A perfect one is exclusive and exhaustive in its alternative prediction. The rules then are as follows :

1. Affirm one of the alternatives in the minor, and deny the other in the conclusion. Reason : because, the members being exclusive, if one is true the other is false (principle of contradiction).
2. Deny one of the alternatives in the minor, and affirm the other in the conclusion. Reason : because, the members being exhaustive, there is no middle between them ; hence if one is false the other is true (principle of excluded middle).

The perfect disjunctive syllogism has therefore two moods : one affirmative,—*Modus tollendo ponens* ; another negative—*Modus ponendo tollens*. Example :

Affirmative mood :

Bodies are either at rest or in motion,
This body is not at rest ; therefore,
This body is in motion.

Or

This body is not in motion ; therefore,
This body is at rest.

Negative mood :

Bodies are either at rest or in motion,
This body is at rest ; therefore,
This body is not in motion.

Or

This body is in motion ; therefore,
This body is not at rest.

As regards the imperfect disjunctive proposition, the members, not being precisely opposed, may be true together. Consequently there is only one mood of inference ; if not one alternative, the other—*modus tollendo ponens*. This is the least important result that a disjunctive proposition is supposed to have. For instance :

The author of the false accusation is either ignorant or malicious. But he is not ignorant ; therefore, he is malicious.—As he may be both, I cannot argue : But he is ignorant ; therefore, he is not malicious.

Such disjunctive syllogisms are used, and there is no reason for rejecting them. Let it be understood that the alternatives must be complete or exhaustive either in the particular universe of the speaker, or as a matter of fact. Otherwise the disjunctive proposition is unreliable, from which no valid inference is possible.[1]

A disjunctive proposition, when the members are exclusive, may be stated as a conjunctive one, expressing the simultaneous opposition of two alternatives.

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[1] See the Import of Disjunctive Propositions, Chap. V., No. 7.

The inference then would be: if one, not the other (Principle of Contradiction). For instance:

No one can at the same time be a republican and an imperialist. But democrats are republicans; therefore, they cannot be imperialists.

As a matter of exercise, a disjunctive syllogism may be changed into hypothetical or categorical. Thus the rules of the former are compared and checked by those of the latter: and one sees the unity that exists among our various reasonings.

10. Dilemma. This is a hypothetico-disjunctive syllogism. The major premise is a double hypothetical proposition; the minor is a disjunctive proposition; and the conclusion may be categorical or disjunctive. The moods of the dilemma are the same as those of the hypothetical syllogism. One is affirmative, in which the minor affirms the antecedents and the conclusion affirms the consequents; the other is negative, in which the minor denies the consequents and the conclusion denies the antecedents.

Each mood admits of two varieties, namely, simple and complex. In the simple the conclusion will be categorical; for the obvious reason that the hypothetical propositions have a common antecedent or a common consequent.

Example 1. Simple affirmative or constructive. Two hypothetical propositions with a common consequent:

If you are operated upon, you will die soon; if you are not operated upon, you will also die soon.

But you must either be, or not be, operated upon; therefore (in either case):

You will die soon. (The conclusion is a simple categorical.)

Example 2. Complex affirmative or constructive. Two hypothetical propositions with double antecedent and double consequent :

If I remain on board, I shall be burnt ; if I jump overboard I shall be drowned.

But I must either remain on board or jump overboard ; therefore I shall either be burnt or drowned. (The conclusion is disjunctive.)

Example 3. Simple negative or destructive. Two hypothetical propositions with a common antecedent :

If stealing were lawful, it would not be punished by God, nor punished by men.

But stealing is punished either by God or by men ; therefore stealing is not lawful. (The conclusion is categorical.)

Example 4. Complex negative or destructive. Two hypothetical propositions with double antecedents and double consequents :

If he were clever, he would see his mistake ; if he were honest he would acknowledge it.

But either he does not see his mistake, or does not acknowledge it ; therefore,

He is either not clever or not honest. (The conclusion is disjunctive.)

The structure of the dilemma is not changed by laying down the disjunctive proposition first. This is often done, and the argument seems more effective.

A dilemma may consist of more than two hypothetical propositions, and the alternate or disjunctive members may be more than two. Then it becomes a trilemma or polylemma. Example :

This man fails in his duties either for lack of knowledge, or from habitual distraction, or out of malicious will. If it is for lack of knowledge, he is unfit to hold the office ; if it is for habitual distraction, he is equally unfit ; if it is out of malicious will, he is still more unfit. Therefore in any case this man is unfit to hold the office.

The nature of the dilemma, though variously explained, may be concisely expressed in the following definition :

A syllogism in which we argue, or infer the same conclusion from each of the alternate members that are possible in a given point at issue.[1]

Rules of the dilemma :

1. It is essential that the alternative members of the disjunctive premise be complete or exhaustive. An alternative omitted or overlooked affords an escape from the conclusion, and the whole reasoning becomes invalid.
2. The consequents must universally follow from the antecedents, and must be indisputable ; otherwise the dilemma may easily be retorted, or used to draw an opposite conclusion.

Example faulty against Rule I. (A proof that people cannot be governed.)

If people are governed by force, they rebel ;

If people are governed by reason, few listen to it.

But they must be governed by force or by reason ; therefore,

People will either rebel, or few will listen (people cannot be governed).

The alternatives are incomplete in the minor. The missing member is : or by a wise combination of both force and reason.

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[1] The uses of a Dilemma may be (a) *Intellectual* (to decide between rival hypotheses) ; (b) *Practical* (to make a decision) ; (c) *Rhetorical* (for a disputant to overthrow his opponent). For up-to-date illustrations, see Welton, *Groundwork of Logic*, pp. 234-40.

Another Example :

People are poor either for want of employment, or for unfitness to work.

If poor for want of employment, they are not to blame ; and if poor for unfitness to work, they are not to blame either ; therefore,

Poor people are not, in any case, to blame for their poverty.

The fault is in the alternatives not being exhaustive, as there are other reasons why people can be poor.

Example faulty against Rule 2. (The Athenian mother persuading her son not to enter public life.)

My son, in public life you will act justly or unjustly. If you act justly men will hate you ; if you act unjustly the gods will hate you ; hence, my son, do not enter public life.

The son replies that he must enter public life, for

If I enter public life, I will act either justly or unjustly. If justly, the gods will bless me ; if unjustly, men will bless me ; therefore, I must enter public life.

Another Example : (Teachers are of no use.)

If teachers are ordinary, they cannot teach higher knowledge ;

If they are extraordinary, they will not teach lower knowledge.

But teachers are either ordinary or extraordinary.

Therefore, they either cannot teach higher knowledge, or will not teach lower knowledge (or they are of no use).

Rebuttal : (Teachers are useful.)

If teachers are ordinary, they can teach lower knowledge ;

If extraordinary, they can teach higher knowledge.

But they are either ordinary or extraordinary.

Therefore, they can teach either lower or higher knowledge (or they are useful).

The fault is in the major, where the consequents are not indisputable. To rebut the argument, re-state the hypotheticals, denying the previous consequents, or replacing them by different ones.

Observe that to rebut a dilemma is not the same thing as to refute it. In rebutting we render the dilemma of our opponent ineffective, but we do not say where the dilemma fails. In fact both dilemmas may be equally weak or wrong. Refuting, on the other hand, attacks the argumentation in the dilemma and shows that this is inconclusive. Take the following example: "If students are idle examinations are useless, and if students are industrious examinations are also useless; but students are either idle or industrious, therefore all examinations are useless." Here the hypothetical propositions may be true for some extreme cases of idle students, as well as of industrious students, but not regarding the majority. In other words, these propositions are not universally true; hence the universal conclusion does not follow. Another way of refuting a dilemma is by pointing out a member missing in the disjunctive minor premise, as in the examples given above illustrating Rule I. The missing member is not touched by the argument, consequently the conclusion does not follow.

A dilemmatic argument is interesting and popular on account of its sweeping character, and the peculiar position of the adversary between two undesirable alternatives. But dilemmas are often incorrect. It is rather difficult to establish a complete disjunctive and a perfect hypothetical, as required by the rules.

Dilemmas may be expanded into hypothetical syllogisms. Example:

If that man were wise, he would not speak against the sacred books in jest; and if he were good, he would not do so in earnest. But he does it either in jest or in earnest; therefore he is either not wise or not good.

Hypothetical syllogisms:

If that man were wise, he would not speak against the sacred books in jest; but he does so speak in jest, therefore he is not wise.

If that man were good, he would not speak against the sacred books in earnest; but he does so speak in earnest; therefore he is not good.

11. Is the Syllogism a Universal Type of Reasoning? Some logicians maintain that the syllogism is a universal type of reasoning; because any transition from the known to the unknown, from observed facts to an unobserved law, from general to particular and from particular to general, must be by means of a bridge or a common ground, which is the middle term in the syllogism. Hence the syllogism is the test of every kind of mediate inference.[1]

There are arguments, however, quite common, and perfectly evident, and yet not shaped in the three-term form, namely those dealing with relations. Examples:

(*a*) Relation of equality: A equals B; B equals C .... A equals C.

A is a brother of B; B is a brother of C .... A is a brother of C.

(*b*) Relation of degree: A is greater than B; B is greater than C .... A is greater than C.

(*c*) Relation of time: Socrates lived before Plato; Plato lived before Aristotle .... Socrates lived before Aristotle.

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[1] "We can no more reason without making syllogisms than we can speak and argue without forming sentences." Archbishop Thompson, *Laws of Thought*, p. 244. See also Whately, *Elements of Logic*, Book IV. Dissertation on the province of reasoning.

Arguments of this kind, some authors say, are semi-syllogistic, and may easily be reduced to syllogistic form by subsuming them under a general statement of identity. Thus for (*a*) the form would be :

Whatever is equal to B is equal to C,
A is equal to B ;
A is equal to C.

And likewise for (*b*), (*c*) and similar cases.

Other logicians, however, declare that the syllogism is not a universal type of reasoning. They say that arguments like those mentioned, dealing with relations, are not syllogistic. If stated in logical form they do not gain thereby. They stand on their own self-evident inference.

It seems, however, that those arguments are really deductive. They are of a *hypothetical* nature ; that is to say, the major premise is a self-evident hypothetical principle, and the minor premise affirms or denies the fact of the hypothesis. What happens in those arguments is that the major premise is presupposed, and the minor *only* is mentioned. Such arguments in concrete sciences may be certain or probable according to the nature of the premises, of which more hereafter.

12. The Syllogism as a Means of Knowledge. The syllogism is essentially deductive. Owing to this fact, an objection has been raised as to its value for the purpose of increasing knowledge. The difficulty may be stated as follows : The conclusion of a syllogism is contained in the major premise. For instance, in the syllogism : " All men are fallible ; Socrates is a man : Socrates is fallible ", the conclusion "Socrates is fallible ", is included in the major, " all men are fallible." If Socrates is not included, then the proposition is not universal. Generally speaking a universal proposition is nothing more than a sum of particulars. Therefore the process from the major to the conclusion is not an increase of knowledge. Nay more, the syllogism begs the question, as it proves the conclusion by the same conclusion already assumed in the major premise.

The answer to this objection is quite simple, namely, the major premise contains the conclusion implicitly, not explicitly. In other words, the conclusion is contained in the major premise, but this is not known until the minor premise is laid down. In the previous example, Socrates is fallible because Socrates is a man. The syllogism therefore progresses from one statement to another implied in it ; and the process that unfolds this implication is certainly a means of knowledge. It points out the conclusion and at the same time the reason for it—which is a perfect way of demonstration.

We may add that the general proposition is not a sum of particulars, but a relation of identity between subject and predicate. This relation holds universally, not from the enumeration of particulars, but from the nature of things themselves. Thus, for instance, all men are mortal not by complete enumeration of single men, but from the nature of man, a living organism that implies mortality.[1]

13. Inference from Particular Premises. Regarding the syllogism, we said before that two particulars yield no conclusion. But if both premises are quantified by ‘ most ”, a legitimate conclusion follows in the 3rd Figure. For instance :

Most books are novels,

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[1] For further explanation see Chap. XVII., No. 6, on the fallacy of begging the question.

Most books are useless,
Therefore, some useless things are novels.

In the case of definite numbers, the conclusion will be definite also, viz., the excess of the sum of both quantities (major and minor) over the total collection. Example :

Three-fourths of the class were studious,
One-third of the class failed ; therefore, at least
One-twelfth of students were studious and failed.

The validity is made clear by considering that the premises tend to be universal rather than particular propositions. The predicate agrees with all the individuals of the subject in both premises. But the subjects overlap each other ; and, consequently, a portion possesses the predicate of the major and that of the minor premise. Hence this portion, definite or indefinite, stands as a middle term.

14. Probable Deductive Reasoning. A deductive argument is probable (that is to say, it yields a probable conclusion) when either one or both premises are probable. Let us examine each case. First, if a legitimate syllogism has one premise probable and the other certain, the conclusion will be probable. For if it is not certain that two terms agree with a third, it cannot be certain that they agree with each other. Examples :

Splendid crops are a source of income ; (Certain)
That property is likely to bring me splendid crops ; (Probable)
That property is likely to bring me a source of income (Probable).

Secondly, a legitimate syllogism with both premises probable yields a conclusion which has the probability of the major weakened by the improbability of the minor. The reason is because the conclusion partakes of the probability or certainty of both the premises. It is similar to an event the happening of which depends upon two interdependent hypotheses. Each one must be verified in order to turn the event into a fact. Meanwhile the event stands on the ground of the two suppositions. Example :

Graduates of an English University are likely to be employed by Government ; (Probable)

By studying in England I may become a graduate of an English University ; (Probable)

By studying in England, I am likely to become a Government employee (Probable).

Observe that, for practical purposes, the modalities "probable, possible," and the particular quantifications "some, most, many," etc., are equivalent to probability, and come down to the conclusion as a natural inference.

Again, in a chain of probable arguments (Sorites, Polysyllogism, Epichireme) the final conclusion decreases in probability with the number and weakness of probable premises. They are called self-infirmative arguments.

Corroborative arguments, i.e., arguments used independently to prove the same conclusion, bring about a very different result ; that is to say, they increase the probability of the conclusion. A common instance of corroborative argument is that of a number of circumstantial evidences, each bearing independently on the same point to be proved, say the identity of a murderer. Many theoretical propositions in every science are proved by several arguments, each somewhat probable, which when taken together render the proposition highly probable or almost certain.

The rules to estimate the amount of probability in self-infirmative and self-corroborative arguments, will be given in Induction, as those rules rest on material considerations.

15. **Fallacious Arguments** are false arguments disguised under the form of a true one. They are likely to occur in a great variety of ways. Since terms, propositions and inference make up the various parts of an argument, whatever is wrong in them must be detected and disclosed when united into an organic whole for the purpose of drawing a conclusion.

To analyse arguments, therefore, the student must begin by acquainting himself with the formal rules laid down in the course of Deduction, particularly with those concerning immediate inference, the syllogism and kinds of arguments. Thus he will be able to detect a formal **deductive fallacy**, which consists in breaking one or another of the rules of correct inference. A few examples to make this plain :

1. Two negative premises :

No fool is fit for high positions ; all here present are not fools ; therefore, all here present are fit for high positions.— The two premises, being negative, imply no comparison with the middle term ; hence no inference is possible.

2. Undistributed middle :

John Smith is a Mahomedan, for he holds the opinions all Mahomedans hold,—This enthymeme is wrong, as will appear if stated in full as follows :

All Mahomedans hold Mahomedan opinions,
John Smith holds Mahomedan opinions ;
John Smith is a Mahomedan—A syllogism in Figure II with undistributed middle.

3. Illicit process :

Whatever is universally received is true,
The existence of God is not universally received ;
The existence of God is not true (Whately). There is here an illicit process of the major term.

4. A false disjunctive proposition :

The propositions " John is living, John is dead " are both either true or false ; if true, two contradictories are true ; if false, two contradictories are false ; therefore two contradictories may be both true or false.—The disjunctive proposition is incomplete. Two propositions may be both true, or both false, or one true and the other false.

5. A false hypothetical mood :

If a man lies he is either vain or a coward. This man does not lie : therefore he is neither.—A wrong inference ; for he may be vain or a coward for reasons other than being a liar.

Let this suffice for the purpose of practice. The full treatment of fallacies, embracing the logical, semi-logical and material ones, will come naturally at the end of the whole course of Logic.

PART II

THE GROUNDS AND METHODS OF INDUCTIVE REASONING WHICH LEAD TO THE ORGANIC CONSTITUTION OF SCIENTIFIC KNOWLEDGE

CHAPTER IX

NATURE OF INDUCTION

1. Need of Induction. Formal Logic has for its starting-point *certain propositions*, from which we deduce conclusions. But in order to be sure of the conclusions, we must be sure of the truth of the propositions made use of as major and minor premises. The question therefore arises : How can we be sure of these propositions ? Some of them are *self-evident* on a mere inspection of their terms ; but others rest upon contingent facts which can only be ascertained by observation.

Speaking in general, an attempt may be made to prove each premise by another syllogism, and this second syllogism by a third till we reach a definition or a fundamental principle,—a process used in abstract sciences like Geometry. This kind of proof, however, is hardly applicable to synthetical propositions as they are usually found in concrete sciences. A statement in which the predicate is not included in the connotation of the subject must be tested by experience ; that is to say, by a study of nature as it is perceived by our senses. Suppose a demonstration is required for the following truth : "Acid does not dissolve gold." The only way open is to show by experiments that gold is really unaffected by acid. This is what is meant by saying that we need Induction to prove the material truth of propositions.

Hence, before applying Formal Logic to many matters which we have to reason about, we must study the rules and principles on which we can arrive at fact-propositions, and can check the process by which we arrive at them ; in order to be sure that they are the genuine outcome of our observations, and that our observations are themselves correct. But this means a process in a new and contrary direction, a process which we call Induction. Putting it briefly, the purpose of Induction is to build up propositions or to establish generalisations more or less universal, probable or certain, from the knowledge of facts observed.

2. General Definition. Induction is a process of reaching a universal proposition from particulars or individuals. In other words : A method of reasoning in which the mind proceeds from individual instances to general uniformities.

These uniformities are manifold, varying in universality and certitude. Moral universalities are not so certain as those on physical matters. Examples are numerous. From observed facts showing that living beings require oxygen, the conclusion is obtained that Life in general depends on oxygen for its existence. The analysis of water gives H_2O ; hence the generalisation that All water is composed of those two elements in that definite proportion. Quinine has cured malaria in past cases ; therefore, it will do the same in future cases. It has been observed that bad crops are followed by a rise of prices ; consequently the same will happen in the future.

3. Induction and Deduction Compared. We are now in a position to compare the two processes. Deduction means drawing conclusions from propositions already established, as implied in the word *de-ducere*, to lead the mind *out from* a proposition. Induction means drawing conclusions from facts observed in order to establish propositions, from the verb *in-ducere*, to lead the mind *in to* a proposition. To express it graphically :

	fact	fact	fact	
Induction.	leading	in to		From observed particulars to inferred generalities.
	a proposition			
Deduction.	leading	out to		From established generalities to particulars not observed but inferred.
	fact,	fact,	fact	

Thus Induction, coming first, leads from particular facts to general propositions. Deduction, coming second, leads from general propositions to particular facts.

Wherever deductive reasoning proceeds from a synthetic premise, the truth of our deductive conclusion depends on previous inductive reasoning to establish the truth of that premise. We said previously, when treating of the syllogism, that from truth only truth follows logically ; but that a conclusion can sometimes (*per accidens*) coincide with truth, even though in Logic it be drawn from premises which are false. To avoid this fallacy we must have premises duly obtained from facts by induction. When this is so, the conclusion arrived at is grounded on the premises and is really proved.

One essential difference between the two processes of reasoning is this : Deduction must begin with the universal, from which the conclusion is explicitly developed.

Induction, on the contrary, must begin with individual instances, because everything in nature is an individual thing separate and distinct from any other.

Another and most characteristic difference between the two lies in the inference itself, which is *formal* in deduction and *material* in induction.

Formal inference means that from premises a conclusion is drawn by following the principles of correct thinking. Inference according to principles is the same thing as consistency. For instance, on the strength of a hypothetical proposition, whoever affirms the antecedent must affirm the consequent, but not vice versa. So also whoever denies the consequent must deny the antecedent, but not vice versa. Briefly, deduction is concerned with the conditions of formal truth.

Material inference, on the contrary, is concerned with the conditions of material truth. Material truth means that the facts themselves, expressed by subject and predicate in a proposition, are in nature related as they are said to be. To investigate this material truth implies an analysis of the denotation of the subject, finding out whether each individual possesses, or does not possess, the quality of the predicate. In short, Induction deals with things, while Deduction remains in the order of mental processes. Observe, however, that the one process is not independent of the other. Deduction presupposes true propositions in a material sense, therefore presupposes induction. On the other hand, induction cannot pass to a generalisation without a ground, or a mental conception binding together all the individuals of the subject to the quality of the predicate. Such a principle is universal and presupposed. An inductive conclusion, therefore, cannot become universal without the aid of deduction. Of this we shall see more later on.

4. **Induction and the Syllogism.** The syllogism leads to a *certain* conclusion on the strength of formal inference ; but induction in most cases is not more than *probable.* For this reason inductive arguments are often stated in the form of enthymemes, in which a generalisation is supported by a particular premise. The form is convenient, and is commonly used to express mere probabilities, as Aristotle himself shrewdly observes.[1] From this it will be seen that Induction is something very different from Deduction. The two ways of reasoning may be contrasted as follows :

Deductive reasoning :	Inductive reasoning :
All books of Scott are historical novels,	a, b, c, are historical novels,
a, b, c, are books of Scott,	a, b, c, are books of Scott.
a, b, c, are historical novels.	Books of Scott are historical novels.

Observe (1) The order of propositions is reversed in the two processes, and so are the middle and minor terms. (2) The conclusion in inductive reasoning is formally particular. In fact it can never go beyond the number of individuals examined. (3) But if the particulars a, b, c, be a complete enumeration, the minor premise becomes a U proposition and the conclusion is universal. Example :

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[1] Mellone writes : "The Aristotelian Enthymeme is of great logical significance ; it covers the elementary forms of what later writers have called Induction. And in his treatment of it, Aristotle marks some of the stages by which we pass from guess-work towards scientific knowledge." After which he proceeds to show by instructive examples how the various kinds of Enthymemes suggest general rules. For example : "This man fled from the scene of a certain crime ; therefore, he may be the murderer." The general rule suggested is "Murderers flee from the scene of the crime." (Mellone, *An Introductory Text-Book of Logic,* pp. 251-60.)

a, b, c, are Brahmins,
a, b, c, are all the members of the Sanskrit class,
All the members of the Sanskrit class are Brahmins.

This, being an A A A argument in the third figure, may be expressed in the mood Barbara thus :

a, b, c, are Brahmins,
All the members of the Sanskrit class are a, b, c.
All the members of the Sanskrit class are Brahmins.

This type of reasoning is called by ancient writers perfect induction. No doubt the conclusion is certain and universal. But the name of " perfect induction " is hardly deserved. For this sort of induction is very limited, as we can seldom gather all the individuals of a kind ; and besides, it is hardly an induction at all. By examining all the individuals we arrive at nothing new, except a concise expression embracing them all ; and such a generality is not useful for all similar cases. It explains only one case.

5. Induction by Simple Enumeration. Summing up, we reach the definition of enumerative induction : The process of inferring a conclusion on the strength of number of individuals examined ; universal and certain, if all are examined ; particular or probable, if only some are taken into account.

6. Central Point of Induction. The number of individuals examined is of little value for the purpose of science. Our endeavour is to find out another ground, a connection between a few individual facts and a general truth ; so that what is said of a, b, c must also be said of all similar cases, past, present and future. The inference cannot be a leap in the dark, but must be made according to principle.

All rules and canons of Induction are directed to the solution of this problem. Let us take an instance: "All life depends on oxygen." The problem is to arrive at this universal proposition. The predicate "oxygen" is not included in the subject. Proceeding by experience, we shall find out how living things decay and die without oxygen. As experience is limited to some individuals, a reason or a law is needed binding together all living beings to oxygen. To discover such necessary connections hidden in the individuals is the constant effort of scientific men.

7. **Scientific Induction** may be thus defined: A process of observation and experiment by which the mind discovers in the facts analysed certain relations that involve universality. All such relations are comprehended under the name of Uniformity of nature; they are also called Laws of nature, or Causations. Observe that the number of indications, or the amount of evidence obtained regarding the existence of a general connection, is precisely what makes the conclusion either more or less probable, or certain. Experience merely gathers the facts. It is for the mind to estimate how far the action, the quality or the attribute is rooted in nature. The reasoning then may be expressed as follows:

a, b, c, are cases of malaria,
a, b, c, are cured by quinine;
Quinine is a cure for malaria.

Natural connections are universal connections,
Cases of malaria are naturally connected with quinine;
All cases of malaria are cured by quinine.

This ground, that nature is uniform in its operations (or that like effects are produced by like causes, or that the same cause in similar circumstances will produce the same effect), is a postulate of Induction. To ascertain that a given phenomenon is naturally connected with another, is an insight of the mind depending on the number of instances, their resemblance, and above all, precision in observation and experiment—a work that may last many years and occupy many observers.[1]

That nature is uniform, or determined towards one and the same effect, is the ground of Induction. Yet nature is often complex ; and this determination in practice is not always without exceptions. The ground is different in different sciences. For instance, the laws of prices, wages and rents in Economics, the law of natural selection in Biology, the law of personal interest in Sociology, are not of the same value as the law of gravitation in Physics, the law of limits in Mathematics, the law of proportions in Chemistry.

8. **General Survey of Induction.** Here it will be well to sketch out the whole process of Induction. The first step is to conceive a clear and complete idea of the facts themselves, noting down all the details available. [This subject will be fully developed in the chapter on observation and experiment.]

The second step is to think out all possible causes or explanations of the facts observed, fixing the mind upon the most probable one and adopting that as a hypothesis. [This point will be considered in the chapter on hypotheses.]

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[1] Bacon was right when, speaking of that peculiar power of observation, he wrote : " Ars experimentalis sagacitas potius est, et odoratio quaedam venatica quam scientia." The art of observation is a kind of sagacity, or scent, rather than a science. (De Augm. Scient. L. 5, c. 2.)

The third step is the verification of our hypothesis, testing it in various ways so as to show that it is a fact. The work is done according to the methods of induction, with the result that a hypothesis becomes a law. Once this work is accomplished it is easy to fall back on deduction, and to explain the events or facts of nature by the established laws, which have now become universal premises.

Here a distinction should be observed. In studying the process of induction, we are not pursuing the art of discovery. To find out a law is something quite different from looking at the process followed. Discovery depends on talent, knowledge and experience ; and none of these are imparted by Logic. Our sole present object is to examine the methods followed in scientific research, in order to estimate their conclusiveness or validity.

9. **Historical Sketch of Induction.** The Logic of induction has developed greatly with the increase of scientific research, and is growing every day with the new attempts to solve problems in Sociology and natural sciences.

Aristotle (384-322 B.C.) and after him the Schoolmen, treated of induction by simple enumeration, and suggested that the mind has the power of perceiving a general law in individual facts.

As far back as the thirteenth century we find Roger Bacon, a Franciscan monk, who died at Oxford in 1294. He not only suggested and encouraged, but excelled in experimental methods, inventing instruments for that purpose.

Francis Bacon (1561-1626), an English statesman and philosopher, is honoured with the title of " Master of Induction." His writings, particularly his *Novum Organum*, stirred up a movement towards the discovery of nature. About the same time a number of scientific men: Leonardo da Vinci, an Italian (1452-1519), Nicolao Copernicus, a Russian (1437-1543), John Kepler, a German astronomer (1571-1630), Galileo, an Italian (1564-1642), and several others devoted themselves with great success to the study of natural phenomena.

Isaac Newton (1642-1727) sketched the real foundations of induction, namely, observation and verification of hypotheses.

John Herschell (1792-1871) in his *Discourse on the Study of Natural Philosophy*, laid down the methods to be pursued in scientific investigation.

William Whewell (1794-1866) wrote extensively ; his main works are : *History of the Inductive Sciences* and *The Philosophy of the Inductive Sciences.*

John Stuart Mill (1806-1873) aimed at a reconstruction of the whole system of Logic based entirely on the process of induction in its various forms, which he distinctly recounts and specifies. The so-called direct methods of induction are most carefully analysed and inculcated as a means to attain material truth.

William Stanley Jevons (1835-1882) explained the nature of inductive reasoning in his book : *The Principles of Science.*

Much remains to be done to establish scientific order in all modern branches of knowledge.

CHAPTER X

FORMAL GROUNDS OF INDUCTION

1. The Two Grounds. As explained before, true inductive inference—that which leads to a universal conclusion without requiring exhaustive enumeration—is based on two grounds: One ground may be called material, consisting of facts which are the object of sense-perception, observation and experiment. The other ground, namely, the general connection between phenomena, is formal, and belongs to the mental order as a principle or axiom.

How the mind comes to be in possession of such general truths is theoretically discussed. At present it is sufficient to state that they are formal grounds of induction, in the sense that they are axioms, the truth of which is in practice admitted by all. In fact, they are applied in concrete cases whenever the mind studies phenomena, in order to draw from them a general conclusion.

2. Uniformity of Nature. Let us begin with the most general of these principles, that nature is uniform in its operations. All objects of perception are individual things, each standing by itself, and presenting an immense variety as to character and activity. A short spell of experience shows us that in spite of this variety, there is a good deal of repetition of the same kinds of being and the same kinds of action; and the same events recur again and again with such regularity that we instinctively expect them to occur again as before. As this expectation verifies itself, we gradually get into a complete confidence that the future will be a continuation of the past; that the sun which rose to-day will rise to-morrow; that the seasons will succeed each other this year as they did last year; that fire which has burnt me already will burn me again; and so with hundreds of other experiences. In this way we reach a conviction about the general uniformity of nature—not only along certain lines which are manifest to us, but also on other lines which have not yet come under observation. At length this conviction becomes a universal conception, and assumes the character of a law dominating the whole of the universe. This conviction, expressed as a principle, is the foundation of experimental science.

3. Various Uniformities. The uniformity of nature, taken in a general sense, may be defined: The fixed relations existing in nature between things and things, or between phenomena. Or again, the sum of the laws regulating the course of the world. Such relations and laws are numerous, and as various as the things themselves. We may classify them under the following heads:

(*a*) The law of Causation, and other general principles assumed by the various sciences.

(*b*) The law of Co-existence, according to which properties and inseparable accidents are invariably attached to the kind of things they belong to. This law embraces many generalisations laid down in Physics, Chemistry and Biology, and are the basis of classification.

(*c*) The uniformity of time and space, in the sense that these realities are subject to a common measure; for example, one hour is equal to another, and one mile is equal to another, etc.

(*d*) The unknown will resemble the known. This presumption stands as a support for probable reasonings based on statistics, which are made up by simple enumeration.

(*e*) Things similar in one respect are likely to be similar in another. This is a presumption peculiar to arguments from Analogy.

The generalizations (*d*) and (*e*) are not laws of nature so far as we know. Still we are inclined to use them as suppositions or hypotheses that serve as grounds of induction. The conclusion cannot go beyond probability, because the ground is not fully ascertained.

4. **Meaning of the Uniformity of Nature.** The uniformity of nature is a general belief of mankind; but how do people come to believe in that uniformity? We are all accustomed to see that from certain conditions a certain effect or phenomenon invariably follows. Hence the belief that the same will happen in the future. Accordingly the uniformity of nature is expressed by a hypothetical proposition: If **X**, then **Y**; in which **X** stands for a given set of conditions, and **Y** for a phenomenon that follows invariably from these conditions. If the electric current is turned on, light or heat or motion will follow. Experience, therefore, accounts for the general habit, recognised by all people without exception, of looking upon the operations of nature as uniform.[1]

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[1] Here is an appropriate passage from Shakespeare:

Touchstone. Hast any philosophy in thee, shepherd?
Corin. No more but that I know that the property of rain is to

This explanation is satisfactory as far as it goes. But the leaders of the sensist school (Hume, Mill, Bain) stop here, and regard the uniformity of nature as merely a mental habit based on repeated single experiences. According to them the belief is subjective, and there is no law or necessity corresponding to it in nature. To this we reply as follows :

Experience is certainly a factor that awakens the intellect and inclines it to believe. To add, however, that no law or necessity corresponds to it in the constitution of nature is objectionable for the following reasons :

First, experience alone does not explain why we believe in certain uniformities more than in others. A man will believe that " all crows are black, all boys are playful, all people look to their own interest ", with an assurance far weaker than that which he feels in believing that " All liquids seek their level, the sun will rise to-morrow, a mango seed will grow into a mango tree." And yet experience is, or may be, the same in both cases.

Secondly, a hypothetical proposition expresses a necessary dependence of the consequent upon the antecedent ; in other words, the connection holds universally. Now experience is limited to a few past cases, and to draw universality from them is illicit ; on the other hand, to infer a connection limited to those cases does not bring us to a uniformity of nature. In brief, any attempt to prove the universal from experience is begging the question ; for it implies the argument that

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wet, and fire to burn ; that good pasture makes fat sheep ; and that a great cause of the night is lack of the sun.

Touchstone. Such a one is a natural philosopher.

(*As You Like It*, Act III., Sc. II.)

a few cases reveal a uniformity in nature because nature is uniform.

We must try elsewhere for the solution of this question. The fact of the belief is attested by the consciousness of everybody, philosophers and non-philosophers. To account for this belief we must look to the constitution of our mind, which has the power to perceive in the nature of things the character of universality. We can argue that just as the intellect extracts the effect from the cause, the particular from the universal, or sees the connection that exists between two propositions ; so in like manner the same intellect, by a power of intuition, brings to light the unseen uniformity and the unseen causation that lies hidden behind the fact, for instance, that fire burns. The inference that fire is the cause, and that fire will always behave that way, is easy and convincing ; because there is plan and finality in the efficient cause, and finality means stability. The intuition is consequent to, and helped on by the evidence of the senses. In some instances we know the law ; in other cases the law is unknown, and we substitute instead some hypothesis or presumption—as for instance " The future will resemble the past." Summing up, the uniformity of nature means that there are necessary connections, or fixed and rigid laws in the constitution of nature which are perceptible to our minds.[1]

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[1] Philosophers are divided in explaining how the mind reaches the notion of causation and what the nature of that notion is. For the sensist school of Locke, Hume and Mill, causation is merely a belief of mankind ; according to Kant, it is a subjective category of the mind ; traditional philosophers assert the objective reality of causation, and so does " common-sense " which remains clear and unshaken in spite of subtle speculations.

We may add that the notion of causation as understood by science

5. Causation. We stated above that the uniformity of nature embraces many kinds of regularities or laws recognised in nature, according to our knowledge of it. Of these we single out at present the main one, causation. The law of causation is the main feature of the uniformity of nature ; it is the most important among the various uniformities, on account of its universality, and of its practical use in induction. The universe presents itself as a network of causes and effects, interlaced in a most curious complication, from the lowest to the highest agents. The work of science is to disentangle and unravel the threads : finding out, so to speak, the two ends pulling in one or other direction.

Observe the difference between causation and the attribute of uniformity, common to all the laws of nature. Causation is certainly uniform ; but it implies more, namely, transmission of energy, and transformation of one thing into another. The principle of causation, as understood in the experimental sciences, may be stated in four axioms :

> Every event must have a cause.
> Cause and effect are equal in amount of contents.
> The same cause always produces the same effect.
> The same effect is always due to the same cause.

is not that of Hume. Welton writes : "The doctrine of causation adopted by many writers of the empiricist school in natural sciences is nominally that of Hume, but really not only differs from it but is incompatible with it. Hume's conception of reality is necessarily rejected, for it is at variance with that very fundamental idea of the existence of a material world which alone makes natural science possible. They thus reject Hume's reduction of causation to mere belief or expectation. Causation is not with them as with Hume a 'principle of connexion among ideas,' but one between events in a material world" (Welton, *A Manual of Logic*, Vol. II., p. 14).

We have seen so far: (*a*) that the notion of causation corresponds to reality, or means real activity in the things around us; (*b*) that this notion is acquired by an easy process of mental intuition, or most simple inference through internal and external experience; (*c*) that a cause is supposed to imply the above four axioms. It only remains to declare how this notion works in the process of induction; that is to say, how from the axioms of causation science arrives at a true material inference. As to this last and most important practical point, when the scientist comes to apply these principles to concrete cases, his task is to discover the signs of a cause, by which he ascertains that a given phenomenon is the cause or the effect of another.[1]

6. Scientific Explanation of a Cause. In ordinary life the last condition or event immediately preceding the effect is often taken as the cause. Thus, if a man falls down when jumping off a tram car, it is attributed to a jerk of the car, and no attention is paid to the feeble condition of the person. If a thief breaks into a house, the absence of a watchman is taken as the cause, forgetting that the robbery may have been planned independently of his absence.

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[1] Observe that physical science, proceeding on sense-perception only, limits its attention to the efficient cause of material phenomena, namely, the principle which determines by its own action the existence of a new material thing or phenomenon. There are however efficient causes which are not material; and these pertain to the order of life, and especially human life. Besides the efficient cause, the scholastic philosophers deal with the *material*, the *formal*, the *exemplary* and the *final* cause. For instance, a table is made out of wood, has a definite shape, corresponds to a previous design, and is made for a motive which moved the carpenter to take the work in hand. It is worth noticing that science by neglecting all causes other than the material efficient cause, limits also the explanation of reality around us

The scientific mind, on the contrary, does not conceive a cause as something negative, nor merely as a conspicuous condition ; it goes deeper than the common mind. The scientist investigates what forces or conditions are precisely needed to produce the phenomenon.

The practical definition of an efficient cause is usually stated as follows : The invariable, unconditional, immediate antecedent of the phenomenon which we call the effect. Let us take these four terms in turn :

(*a*) The cause is **antecedent** to the effect, and the effect is consequent to the cause ; not by any break of time, but in virtue of an essential relation of dependence expressed by the hypothetical proposition : " If A, then B." The priority of cause to effect is involved in the very definitions of cause and effect. In regard to time, cause and effect must be together so long as the causation is going on. The process of production may often take days, months or years ; and during the whole time cause and effect are co-existent, as when the water of one tank flows into another. Nevertheless there is certain priority of time in this sense, that the antecedent elements first take time to focus down to the act of causation, and then take more time to produce the finished result.

Observe that a cause and its effect are not isolated in nature. The course of nature is continuous, thus forming many series of causes and effects. Besides, the various series are also related among themselves in a way which, though not fully grasped, results in the unity of the world-system. An instance of that continuity may be noticed in the formation of clouds, the falling of rain, the constant flowing of fountains and rivers; a process which, by co-ordination with other series of causes, begins over again in the formation of vapour, currents of air, etc. Thus manifold uniformities reveal the unity of the universe.

(*b*) An antecedent, in order to be a cause, must be **invariable.** Not every antecedent of an event is a cause; in fact every effect has an indefinite number of antecedents that are in no way connected with it.

"Invariable" means that whenever a certain condition or aggregate of conditions occurs, an effect of a certain kind always follows. That particular relation gives us the idea of the uniformity of causation.

(*c*) "Invariable antecedent" will often suggest a cause, but will not prove it, unless the antecedent is **unconditional.** Day invariably precedes night, and yet is not a cause of it. A flash of light is always antecedent to the report of the gun, without being its cause.

By "condition" is meant any factor or element that helps to, or is needed for, the production of the effect. The condition is positive when its *presence* is needed, negative if its *absence* is needed for the production of the effect. To use a familiar example, when the electric current is switched on, the contact of the poles is a positive condition. In water-pipes the absence of a tap is a negative condition for the flowing of water.

We may now define "**unconditional** antecedent." It means that the set of elements or conditions, positive or negative, is complete for the effect to follow. As no agency is lacking, and no other agency is obstructing, the antecedent is said to be unconditional.

This idea of "unconditional" enables scientists to distinguish the true cause from a mere invariable antecedent; not, however, without difficulty. To find out the unconditional antecedent among many and complicated conditions is a matter of careful experiment. In ordinary affairs it is easy to dispose of the difficulty for one who knows the cause thoroughly. If a fan stops, for instance, or an engine goes wrong, it occurs instantly to the mind what condition is wanting or what obstacle is impeding, and the unconditional antecedent is restored at once.

(*d*) "**Immediate** antecedent," as distinct from unconditional, implies the necessity of the law of nature by which a cause is determined to its effect. Without this determination the antecedent, although complete, might be ineffective. The immediateness of the effect, supposing the law of nature, is always the test of the unconditional antecedent.

In regard to immediateness, it is well to observe that nature is very subtle and profound in its operations, whilst our sense-perception is limited. Hence in many cases, such as molecular changes, results in Biology, or social events, the beginning of the effect will pass unnoticed and is therefore a matter of conjecture.

From what has just been said, it is plain that the signs of a cause are interdependent ; an immediate effect presupposes an unconditional antecedent, and this in turn presupposes an invariable antecedent as a cause.[1]

7. Cause and Effect are Equal. This axiom of causation means that an event, scientifically speaking, is not a new thing altogether, but a change of the cause into the effect. If electric fluid produces light, the effect, light, is nothing else but a change in the fluid. In this respect cause and effect are conceived to be equal, both as to the matter transformed and as to the energy embodied in the effect.

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[1] For further explanation see Carveth Read's *Logic, Causation*, pp. 174-91.

This general axiom is not proved by experience any more than the previous one. It is accepted as true for the purpose of induction. Relying on this principle, those who investigate the operations of nature try to quantify cause and effect whenever possible. Thus, for instance, if two or more elements are combined chemically, the weight of the compound is found to be equal to that of the component elements, which indicates that the matter is not only equal, but perhaps even identical. Energy is in like manner estimated to be equal although in practice it is not easy to establish the equality. Allowance must be made for the amount of energy gone astray in resistance, sound, heat, etc., out of the sum which might be expected in the effect. The analysis of the redistribution of matter and energy involves difficulties that may be studied in treatises on the various sciences.[1]

8. The Same Cause always Produces the Same Effect. This axiom of causation is contained in the two previously explained. Were the same cause capable of producing now one, now another result, there would be no uniformity of nature, and the equality of cause and effect would be a mere chance. The law of causation requires that as the actual working of a cause is determined by a fixed set of conditions, the effect likewise must be fixed (or the same) in accordance with the amount of the antecedent elements.

The equality of cause and effect involves another axiom: that the amount of matter and energy in the universe is constantly the same; for otherwise either the causes would be greater than the effects, or the effects greater than the causes. And if causes were now less, now greater at intervals, uniformity would disappear.

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[1] See Carveth Read, *Logic, Deductive and Inductive*, pp. 180-5.

A corollary of the same principle is that no physical power can either create or annihilate an atom, stop a law of nature or start a new one. Thus everything is presumed to be fixed in the constitution of matter, according to weight and measure, unity and uniformity. Science has discovered a remarkable instance of this universal harmony in the powerful motions of the heavenly bodies, which for centuries remain the same according to the laws of motion and of gravitation.

Observe here, that the law of conservation of matter and energy does not include life in any of its different stages. As life is not a mere physical force, so its appearance and disappearance neither adds to nor subtracts from the sum of the physical universe. Life has a totally different function. It directs, selects, regulates and builds up, so to speak, definite forms of beings with a self-possessed power which controls physical and chemical forces. Life is the commander-in-chief of the army, or the controller of the switch board, or the driver of the engine. The problems concerning the origin and evolution of life, especially of intellectual life, are far from solution as yet, and perhaps entirely out of the field of scientific induction.

For this very reason, that life remains so far a category incapable of experimental analysis, it follows that the marks of a true cause are best ascertained in sciences that can use experiments and calculations, such as Chemistry and Physics. In Biology and Sociology the scientific conception of a cause is applied with proper allowances. The equality of cause and effect cannot be taken strictly ; for mental action, free volitions and feelings are not mechanical. Nay, in social sciences the notions of unconditional antecedent and of equivalence are rarely assumed. Bankers and politicians are not lacking in sharp reasoning when they apply induction ; but, as practical cases in social matters often require quick action, talent and sagacity take the place of experiment in discovering the approximate or real cause of an effect.[1]

9. The Same Effect is always Due to the Same Cause. This axiom involves the controverted problem of plurality of causes. Let it be clear that the issue is not whether two or more causes together can produce the same effect. All admit the existence of complex causes and the corresponding intermixture of effects. A complex cause means several causes working together, and so connected that to separate them is beyond our power. Intermixture of effects is nothing else but a product, the factors of which cannot be traced to separate and distinct causes. The seed laid in the ground develops through different stages. Each degree of development is a complex effect due to several causes ; the action of the sun, humidity, air, etc., which taken together may be called the climate, that is, a complex cause of that compound effect. A photograph is a familiar example of a product where many elements mingle together as a result of a compound set of factors, optical and chemical, concurring in the production. There are various kinds of composition of causes. In chemical composition, for instance, the properties of the effect are other than those of the compounding factors ; in mechanical composition the effect is of the same kind but greater or less than that of each factor.

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[1] For further explanation see Bain, *Logic*, part second, *Induction*, Chapter IV. Law of Causation.

The precise question involved in the axiom under discussion is whether the principle of causation is reciprocal in the sense that, as every cause has its effect, so also every effect has its own cause. If **x**, then **y**, therefore if **y**, then **x**. It is customary in language to say that death is brought about by many different causes ; that a candle may be lit in many ways, which is true enough in the ordinary sense of cause and effect. The effect, however, is not precisely the same, as may be shown by a further and minute analysis, taking into account the adequate cause and the adequate effect in the scientific sense. What happens is that we take more or sometimes less than the precise cause or effect. Very often the agent or the factor that starts the action is called the cause, leaving out of account the group of circumstances which are brought into action by that agent. These surroundings are in reality efficient and part of the cause. For instance, a slight rise of temperature may produce a chemical combination or an immense fire ; the heat with the elements around it evidently make up the totality of the cause. Again a demagogue stirs a mob to a riot by a speech ; his fiery words are an inciting power ; the mob with their passions and circumstances constitute the collocation which is entirely turned into action. The cause, therefore, is not fully ascertained until all these predisposing conditions are realised.[1]

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[1] These examples are taken from Carveth Read's *Logic*, p. 181. On plurality of causes see Latta and Macbeth, *The Elements of Logic*, pp. 291-94.

Sometimes we take more than the scientific cause and less than the scientific effect. A certain food or drink is said to be the cause of infectious fever or even death. Quite true in the common meaning of causation, for in reality the antecedent has been responsible for the consequent. Strictly speaking, however, the cause may not be the food or drink but only something in them, a micro-organism injurious to health. And the effect in turn is more than fever, namely, a particular destruction in the organism with the conditions that are proper to it.

Our statement is that the principle of causation is reciprocal, whenever the facts are minutely known, and by experiment we succeed in securing all the essential conditions that make the adequate cause and its adequate effect. Consequently then and then only can there be, scientifically speaking, no plurality of causes. The reciprocal nature of causation as a principle is not proved by experience, but none the less is the very ideal of scientific research, and is found true in practice. The laws of definite proportions in chemistry are a clear example. Oxygen and hydrogen combine in a fixed proportion to produce water ; and reciprocally, water resolves into those elements in the same proportion. In physics there are many instances in which the reciprocal relation of one energy to another is fully ascertained and expressed in mathematical equation. More often than not it is impossible to attain this ideal, owing to the complexity of cause and effect.

Nevertheless, viewing the question from another standpoint, we are bound to acknowledge plurality of causes in all human and natural productions. To begin with, the individual objects in the course of nature are all causes related to effects. Now, these things possess many qualities by which they differ from one another, for they are isolated and complete by themselves, but there is among them a common quality. Thus it comes about that many of those things can actually produce the same effect, and consequently we have plurality of causes. Whether, for instance, seeds be cast about by the wind or by birds ; whether a crop be damaged by frost, by a storm, by fire or by a volcano's eruption, the effect is practically the same and the causes are many. Likewise in human affairs it is usual to employ various means for one and the same production. A letter may be written by one clerk or another. Goods sent out may reach the same destination in a variety of ways ; and so on. Therefore, considering that people use in ordinary life isolated things, which are not the scientific cause, we must acknowledge on all sides plurality of causes.

The scientific and the common conceptions of cause and effect bear relation to the valid inferences from a hypothetical proposition. The relation between antecedent and consequent is like that of cause and effect. Since many causes distributively can produce the same effect, it happens that granted the antecedent the effect follows ; but a denial of the antecedent does not imply the denial of the consequent, which may still result from another. In like manner, a denial of the consequent is sufficient reason for denying the antecedent, but affirming the consequent gives no right to affirm the antecedent, which may not be this but another one. On the other hand, dealing with a hypothetical proposition that expresses a scientific relation, the invalid inferences disappear. We can argue in any one of the four moods and all are equally valid and true, because in that case antecedent and consequent are reciprocal. In other words there is only one cause for its effect and, vice versa, the effect is due to its own cause. It may be remarked that this exception to the general rules of hypothetical reasonings should afford no confusion, for scientific relations of the kind described are well known and bear the name of laws of nature.[1]

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[1] In connection with this chapter the reader may consult the following books:

John Stuart Mill, *System of Logic*, Vol. I., Book III., Chapter V., on the law of universal causation.

H. W. B. Joseph, *An Introduction to Logic*, Chapter XIX. Of the presuppositions of inductive reasoning: the law of Causation.

Welton, *A Manual of Logic*, Vol. II., Chap. I., Postulates of Induction.

CHAPTER XI

OBSERVATION AND EXPERIMENT

1. Facts the Material of Science. In order to study a definite subject-matter we need the matter itself. In order to put up a building the first step is to collect the materials and then to arrange them for construction ; so likewise our knowledge is built up out of concrete facts lying around us, which have to be collected and then arranged for use by the mind. This is what is meant by the old saying : " Nothing is in the intellect that was not previously in the senses." This does not mean that the intellect makes no advance beyond the sense perception. It only means that the intellect cannot begin to operate except from the starting-point of what the senses have presented to it, in the form of perceived realities. [The existence of our senses and their trustworthiness is here constantly presupposed.]

Facts are the ground of our knowledge from another point of view, viz., the possession of truth. Our mental conceptions must be in conformity with the reality of things, which means perfect adaptability to that reality. The mind postulates sense-perceptions, and these postulate reality. Any lack of conformity will bring about error, and thus it may happen that instead of science one builds up a mere dream. Photography serves for a comparison. A true photo is from the object ; if there be no object, there is no image.

There are various kinds of facts, which give rise to a variety of sciences ; and since the facts are different, so are also the means employed and the methods followed to attain truth. Economics, History, Chemistry, etc. are not all built up by one and the same method.

2. What a Fact is. A fact is a portion of reality independent of the mind, and imposed upon us. This means a concrete and individual piece of reality with its circumstances. As concrete it is rich in contents ; as individual, it is narrow in extension. There are many aspects in one and the same portion of reality. Crystals, for instance, are the subject-matter of Chemistry, Physics, Geology and Geometry, but under different aspects.

We may notice here the difference between a *theory* and a *fact*. The latter is concrete and previous to any generalisation, while the former is a generalised expression of facts. Theories, however, may be regarded as facts in a broad sense, when used for further generalisations. Thus in the course of a proof one takes for a fact the established theory that light is a wave-motion ; or again, one presupposes the relation between the hypotenuse and the sides in a right triangle.

3. Colligation of Facts. This expression means that several facts are brought together in a common relation. For instance, the temperatures of a patient at different times, when marked on paper, result in a curve of a definite period. A navigator, by taking down the various positions along a coast, discovers the existence of an island. Observations of the atmospheric pressure made at regular intervals during twenty-four hours, and compared together, reveal a double oscillation in the weight of the air, normal and very remarkable in the tropics.

A colligation of facts is a kind of induction, more or less easy according to the steps followed in the process.

4. The Nature of Observation. We are about to explain how to gather facts. Everybody observes facts in daily life ; but observation, in the sense we take it now, is not a loose and imperfect way of looking at things. It is rather a precise and exact watching for the purpose of science—and this is not so common and easy as may appear to the untrained mind. A mere look at the surface appearance of a phenomenon is not enough. Deceptions and errors in the first steps of induction are just as frequent as in the process of inference.

Observation is the careful inspection of an object for a definite purpose, or in connection with a system of science. To observe carefully implies attention all through, to every detail of place, figure, number and motion relevant to our purpose ; and attention means keeping thought and reality together. Our aim is the recognition of a fact as it is ; and we trust and believe nothing except upon the testimony of the senses.

We suppose our senses to be trustworthy, and so they are ; yet deceptions are not unusual even in the history of the sciences. Each of our organs of perception is adapted to an object of its own according to definite relations. The eye is good at perceiving colours and light, and not so competent in gauging figure and size, specially at a distance. The ear is adapted to sound, and indirectly to its circumstances. Now see how easily errors come in. Taking a substance to be sweet from its colour is to make a jump from sight to taste ; a tower at a distance may look small when in reality it is quite large ; and a good painting makes a plain surface appear as if in relief. In these and like cases the mistake consists in not reckoning with the relations between the object and its corresponding organ of sense. Again, a landscape scene is tinged with different colours according to the changing conditions of the air ; by looking at things through a coloured glass they appear to be of that tint—the hallucination coming from the intervening element. Another familiar example is a spectator watching the performance of a juggler or a ventriloquist. Unaccustomed to fallacious indications, he seems to hear and see what in reality he has not heard or seen. He makes a fact of his own instead of perceiving one—a fallacy of non-observation.

Occasionally deception takes place by perceiving a real fact, but diminished, or augmented. Previous to Galileo it was taken for granted that falling bodies twice as heavy fell twice as fast. Again, it is familiar how an object immersed in water seems to the eye bent or magnified. Crystals were formerly examined only as to their faces or sides, forgetting a most important factor in Crystallography, the angle. This shows that the testimony of one sense often needs to be supplemented by the testimony of another, to avoid the fallacy of faulty observation.

5. **The Mixing of Observation with Inference.** The organ of sense is adapted only to the perception of reality ; it is for the mind to look into the nature of things below the surface. For otherwise we mix observation with inference, which leads to fatal mistakes. An uneducated person, on seeing clouds or smoke floating in the air, will say that those things evidently have no weight—a wrong inference mixed with the act of seeing. Heat, being perceived as radiating from hot things, was until recent times pronounced to be a caloric

fluid—another inference mixed with the sense of touch. I feel a hand picking my pocket, and by inference accuse the person next to me of attempting theft, when in reality the hand felt is that of another person sitting behind. The expressions "sun-rise" and "sun-set" remind us of the old inference that it is the sun that moves daily around the earth. This suffices to show the importance of carefully distinguishing what we actually perceive from our judgments about the laws of nature, or about causes that do not fall under the senses.

Much caution is required to avoid another unconscious deception, namely, that of moulding our observation in accordance with prevailing emotions, wishes and prejudices. To give a few instances. Demosthenes, while fleeing from the battle-field, saw (or imagined he saw) the enemy upon him, when in reality there was nobody around, but only some briers entangling his coat. A man in anger sees at times a kind of insult in another who simply smiles out of kindness. A scientist is likely to find, through the telescope or the microscope, something that is not in the object, but which he expects or much desires to find there. Similarly an historian who examines obliterated inscriptions may supply the particular meaning that he wishes, and which is not borne out by the writing.

Remembering that our mind is both active and receptive during the course of observation, it is but natural that prejudices should influence the perception of facts. One easily leans towards opinion or authority. The qualities of a speaker, the merits of style in architecture, of the beauties of a musical performance, will produce a better impression on the observer, if he knows beforehand the opinion of experts on the matter. Prejudice may even go so far as to make us see nothing worthy in a person of whom we heard unfavourable reports. A book is praised higher than its merits deserve, because of the authority of the writer or out of unconscious deference to public opinion. A blessing would accrue to the cause of truth if prejudices were not so common in daily life.

6. Observation is Selective. Besides being precise and exact, scientific observation must be selective. The facts of nature are rich, and we may say, inexhaustible ; they bear many attributes and relations. An observer is not concerned with everything at once ; he singles out one aspect only at a time, viz., that one which serves his purpose or his system of science. The agriculturist looks at one aspect of the plant, namely, what makes for its better production ; the botanist looks at another aspect leading to classification. In observing a book, attention may be directed to the material make-up of binding and print, or to the contents and perfection of style, or to its price on the market as compared with similar books from different firms and countries. A natural product, say fruit, examined by a chemist means the analysis of its simple elements in quality and quantity ; a physician looks at its various relations to health and complexion ; a merchant observes how it sells. In short, selection in observation means that each observer chooses and actually studies that portion of a phenomenon which serves his purpose or particular science. Selection becomes more and more necessary with the growth and specialisation of sciences. As these branch out further and further, the facts also must be divided into more aspects, and each has to be examined separately.

7. A Qualified Observer. Observation, as explained, is not a matter of mere curiosity or amusement, but something that makes for true knowledge. Now this kind of observation demands certain qualities in the observer.

In the first place, skill is presupposed ; that is to say, an observer must have learnt how to handle facts presented to him, so as to get value out of them. Nature is deep, continuous and complex ; therefore, if a phenomenon is to be examined properly, it needs very often to be split into elementary facts. This separation unravels the tangle of nature, offers an insight into the inside of the phenomenon, and lays bare a relation otherwise hidden. Nature almost defies us, and the observer should be able to defy nature. To do this, however, needs the experience of his predecessors, which has to be learnt from them. Our senses are first educated by natural development ; beyond that, skill grows out of experience and personal education. To give an instance : We all notice that there is air around us ; but what is in the atmosphere is not realised, unless the elements of heat, humidity, pressure, etc., are observed separately. Again, a fact is better observed by varying the circumstances, which requires ability. Thus everybody feels when the weather is hot ; but to get the true temperature of the air requires that one takes it not in the sun, nor close to a wall, but in the shade and free from near objects. Then he will observe a difference connected with the phenomenon of irradiation.

Together with skill, observation demands knowledge and plenty of it. Discoveries are often made by chance, but never by an ignorant man. An up-to-date observer should know the progress made in his department of science, the difficulties encountered and the problems awaiting solution. Such equipment affords him a power to see into the secrets of nature, and a light to distinguish relevant material from irrelevant.

Next to thorough knowledge, interest is required as a motive to push forward one part or other of the investigation, and to follow up step by step the process of observation. Interest is a spur to devise means, and to overcome impatience consequent to a long watching over the slow course of nature.

Above all should be mentioned among the qualities of an observer a sincere love of truth. This must pervade the whole of his observations, in order to insure sound results, and particularly to avoid contradictions ; because truth is always the same for everybody.

In conclusion ; as a musician alone is able to perform on a musical instrument, so only the expert will handle properly the facts of nature. A fact is offered to many ; but its impressions are not received, much less combined in the same way by all. Only a man fully equipped is able to make as much as possible out of a given phenomenon.

8. **The Use of Instruments.** We mean here instruments for the purpose of observation, namely, devices intended to increase the power and efficiency of our senses. The eye, for instance, is limited in its perception of reality ; but a microscope will make accessible what is invisible to the naked eye ; a telescope will reach objects far away ; a seismograph will bring to our notice slight motions of the earth. Thus instruments greatly enlarge the field of observation. Many facts, and even regions of nature, could never be explored without the help of instruments. The conditions of the

atmospheric regions are ascertained by balloons; electric currents and magnetic charges are gathered by generators; and the depths of the ocean are reached with sounding devices. Instruments, again, afford us means of precision and exactness. The hand estimates vaguely the heat of things touched, but a thermometer takes it precisely, and also measures the amount.

Instruments, therefore, are powerful means of observation. Observe, however, that in proportion as that power increases, an increase of ability is required on the part of the observer, and the dangers of bias and error are multiplied. Two observers will not easily give the same account of a complex phenomenon highly magnified under an instrument. Skill, knowledge and practice are very specially presupposed in using instruments aright.

9. **Experiment.** Instruments make a valuable transition from observation to experiment, for they serve to analyse nature below the surface. They are useful even in simple observation, in which a phenomenon is taken as presented to us in the ordinary course of things; but they are absolutely necessary for experimenting. Experiment is observation under artificial conditions introduced by us and forced upon nature. In experimenting we study a product intentionally arranged for a definite purpose. Nature we study at all events, that is to say, the elements and the working of nature; but the fact observed in this case is one which either never occurs by itself, or at least not in that form. A body lighter than water floats in it; but an experiment will show that the weight of the water displaced is equal to the weight of the body. Bodies fall with equal velocity, not apparently, but in reality,

as demonstrated by an experiment in which bodies of every different weight and bulk are let fall in a glass tube without air. Water freezes naturally; but we can freeze it with a solution of salt or alcohol, and notice the difference. In experimenting we combine, separate or apply things in artificial ways; hence it is plain that experiment is much more powerful than mere observation to discover the properties of things and their various relations.

A negative or blind experiment, furthermore, will establish the reciprocal of causation. A blind experiment consists in showing that in the presence of a certain condition or element the effect does not follow. In this way conditions may be excluded one by one as irrelevant to the effect; from which we conclude that what remains is the true cause. The beginning is a relation like this: "If **A**, then **B**"; in which the antecedent is usually a rather complex object to be analysed into factors. Breaking up **A** into recognised or supposed elements we have: **A** = **a, b, c, d.** Now a series of blind experiments will establish the following propositions:

If **a,** then not **B**; if **b,** then not **B**; if **c,** then not **B.** Consequently we should have: "Only if **d,** then **B,**" which is equal to: "Always if **B,** then **d.**" The reciprocal in practice is a difficult task. To ascertain that **a, b, c** are not connected with **B** at all, means exactness in the experiment. And the conclusion: "Only **d** is **B**" is still more difficult. For more often than not a residue **R** remains doubtful, or incapable of further analysis, and the reciprocal is not more than probable.[1]

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[1] For further explanation the reader may consult Bosanquet, *Logic*, Vol. II., Chapter IV., Scientific Induction by Analysis.

It is indifferent for the purpose of finding a relation whether we analyse the effect or the cause. It all depends upon what is known and what unknown. It was known that each pendulum has its own time-oscillation. The unknown cause was analysed by changing the same pendulum in various ways till we came to know that time-oscillation is related to length. Analysing the effect of gravitation that bodies sink more or less in water, we discover their density or specific gravity. If a disease is analysed we succeed in tracing it to a cause, say a micro-organism. In such cases the reciprocal of a hypothetical proposition is realised.

10. Rules Guiding Observation. Summing up, we may state the following rules of observation :

(*a*) Take in the true fact, free from bias, emotions and unconscious inference.
(*b*) Observe what is relevant, completely as to details of place, motion and circumstances.
(*c*) Divide the phenomenon into elementary facts, so as to find out their proper and distinct relation.
(*d*) Isolate your phenomenon by changing, as far as possible, surrounding elements.

11. Observation and Experiment Compared. There is no opposition between the two, but only a difference of form and degree. Practical investigation along the large field of the various sciences is carried on by a gradual transition from observation to experiment. The degrees may be called pure observation, natural experiment and pure experiment. Pure observation means watching a phenomenon happening in the course of nature. In natural experiments the observer takes advantage of circumstances and positions to make out the element of his intentional selection. For instance, exploring the higher atmosphere by means of balloons, the observation of plants in hot-houses, testing by measurements the density of a body, are natural experiments. But pure experiment means interfering with the ordinary ways of nature.

The advantages of experiment over observation are great. In favour of observation we can say that it is easy, within the reach of many, and is very often the only means at our disposal ; but it does not carry us far into the secrets of phenomena. Besides this, we have to wait on nature, which is often very slow and at times too fast. Hence sciences like Geology, depending on observation, make little progress. But experiment enables us to unravel and disentangle the complex facts and thereby to discover hidden relations. Furthermore, we can repeat the experiments or vary them in the laboratory till we arrive at satisfactory results. Thus experimental sciences like Chemistry and Physics are advancing by leaps and bounds.

12. **Oral Testimony.** Many facts are received only from hearsay ; and this opens a new field of observation which involves moral considerations.

Oral testimony is an information given by a witness of the facts—meaning by witness one who got the facts by his personal observation. In order to attain truth, the contents must be referred to reality ; and this implies a criticism of human testimony.

Let us suppose first that we hear well, and understand what is conveyed to us through oral intercourse. The points to be attended to are : (1) That the witness has not been deceived, (2) nor deceives us. It depends on his knowledge and sincerity, whether facts can be reconstructed in our mind as they happened in reality.

That a man knows the facts may be ascertained :

(*a*) From the means at his disposal and from his particular skill, profession or experience.
(*b*) If he be a person without prejudices or illusions on the matter.
(*c*) If there be many who relate substantially the same thing.

Sincerity is recognised :

(*a*) When no motives can be discovered in our narrator inducing him to lie.
(*b*) When no signs of exaggeration are noticeable.
(*c*) When the facts reported can be ascertained by other means, so as to refute him if wrong.

13. Written Testimony. Besides hearsay evidence there is information of facts transmitted in books, monuments, inscriptions, coins and medals. The critical observer will best attain truth if he examines :

(*a*) Whether the authors were in a position to know exactly the facts.
(*b*) Whether the documents are genuine as to the supposed author and date.
(*c*) Whether the documents have been preserved without alterations and additions.

Proper research into those conditions is often a very long and difficult task, as it entails a comparison of books with quotations made at different times, and with references in them to prevailing customs, and many other considerations. For instance : Facts omitted in an historical document, which the author ought to have known and mentioned, are a negative argument against its authenticity.

14. Importance of the Testimony of Others. In most cases we cannot know the facts for ourselves ; we must have recourse to the testimony of others. Distance of place and time, and our ignorance of many subjects make it necessary for every one to depend on authority This is the only source of information in early age ; grown up people cannot ascertain by themselves either distant events or scientific truths, and their only way of overcoming difficulties is consultation, or relying on the experience of others. In short, the range of direct experience is small, compared with the amount of information received from others. Hence the importance of careful inspection and verification of all hearsay evidence and written testimony.

15. Tradition. Some facts are handed down to us by a continuous oral transmission from generation to generation. This source of information is the weakest of all, but not useless. The truth is apt to be distorted in passing from mouth to mouth, and errors can accumulate. The marks of verification are, that the originators were in a position to know the truth ; that the tradition has been constant, and maintained by many people, even by a whole nation or nations ; that it is uniform as to the main facts, and in conformity with contemporary documents. Traditions should neither be wholly rejected, nor wholly admitted. They are at least valuable as indications of customs and modes of life prevailing at their source.

16. Consciousness. We have been speaking of observations that refer to objects outside of ourselves. Another field of observation lies within us, and comprehends feelings and states of the mind. These facts are known to us only by the testimony of our consciousness. The consciousness here referred to is not the act of reflecting upon ourselves, which belongs to the intellect. It is rather the innermost sense by which we are aware of our own thoughts and feelings as they occur. By this kind of consciousness our own internal experiences become facts to us, because we perceive them as part of ourselves just when they occur.

This consciousness is really identical with the act itself that it reports. When the living agent, v.g., feels, this feeling and the consciousness of it are one and the same act. The same holds of thoughts and volitions. Using a comparison, we may say that a living act is a light that illumines itself and also the object. As no other light is required in order to see a light; in like manner, no second act is required in order to be conscious of the first; for then a third act should be required to be conscious of the second, and thus we should require an endless series. In short, if our acts of knowledge did not reveal themselves to ourselves they would never be revealed at all. We can say more than that; a living act is at once conscious of itself, its contents, and the outer object it represents.

From the above exposition it follows that consciousness must be valid in all its acts; no sceptic can gainsay his acts of consciousness. Whatsoever a man is conscious of, that he knows to be true. Extending this principle to every perception, we come to the conclusion that everybody is certain of whatever objective truth is presented to the mind with perfect evidence.

Direct consciousness is strengthened by reflex consciousness, to which an appeal is made to make sure that no illusion or false perception has distorted the natural course of the living perception. Cases of insanity or sickness may happen, but they are easily excluded. The danger lies in mixing up our real perceptions with their causes or circumstances, thus going beyond the object perceived.

17. Memory. Finally our memory is another source of information. It is a store-house of facts lying latent, ready to be called forth at any time. How far memory is trustworthy may easily be tested by any one at different stages of his life. It is a prudent policy not to rely on memory beyond the actual power of reminiscence. To store facts according to their natural relations of time, place and causes is a good means to retain them more faithfully.

CHAPTER XII

HYPOTHESES

1. Where Hypothesis Comes in. Remember what induction is, namely, an inference the grounds of which are facts obtained and postulates presupposed. The place of hypothesis in this process comes next to observation or together with it, as a further step and preparation for inference. Hypotheses are tentative proofs, like preliminary heats preceding a real game.

No rules can be given for starting hypotheses ; they are spontaneous interpretations of facts, that is to say, an original work of the mind. The ability to start hypotheses is a personal endowment, the gift of inventiveness. Previous knowledge and the indications of nature gathered by observation and analogy stimulate the activity of the mind.

2. Definition of Hypothesis. Every supposition we make, even in ordinary life, to account for any event whatsoever, is a hypothesis. For instance, on seeing a person lying on the ground bleeding and senseless, instinctively it occurs to one that a personal enemy, or perhaps an accident has brought about his pitiful condition. The assumption is put forward as a probable explanation of the fact drawn from some kind of evidence.

Mill's definition : " An hypothesis is any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real ; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be, or at least is likely to be true." [1] Observe here that the supposition is built on insufficient evidence, but meanwhile it guides our further inquiry. All that has to be done is to test the hypothesis by facts—a long and patient work, but very often the only way to inductive inference. The function of a hypothesis is to lead us along the course of investigation.

There are degrees in the value of a hypothesis, which vary as the amount of evidence in its favour changes. As the process of verification proceeds, some hypotheses remain stationary, some are gaining or losing ground, and many are dropped out. For instance, the hypothesis of Evolution from a few primitive types of life, to explain the large number of species, has been modified and still remains ; the Atomic theory is constantly gaining ; the Wave theory of light took the place of the Newtonian or Emission theory, and this is being superseded by the Electro-magnetic theory ; the hypothesis that heat is a caloric fluid has been abandoned long ago.

A hypothesis may possess any degree of probability from being merely possible to becoming an established law. The name **theory** is given to a hypothesis sufficient to account for most of the facts to be explained.

3. Kinds of Hypotheses. A hypothesis of cause is intended to find out the origin of a phenomenon wholly or in part. the moving power or the collocation. The atomic theory is meant to solve the problem of the constitution of matter. The tectonic theory of earthquakes points out the cause of these dreadful phenomena. The hypothesis that ether fills all space provides a medium for the propagation of light. A revolution or a strike may be explained by a hypothesis that the leader made a secret alliance for his moral and financial support.

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[1] Mill, *A System of Logic*, Vol. II., p. 8.

A hypothesis of law refers to the manner in which a cause already known is acting. For instance, it is known that micro-organisms are the cause of fever ; but, how they bring about the morbid condition in the organism is a matter of hypothesis. It is a fact that gravitation is a force ; the ratio according to which it acts, namely, in proportion to the product of the masses and the inverse of the square of the distance, is a hypothesis of law. How to make the best use of money in hand is a matter of hypothesis concerning the mode of action, for we know already that capital produces interest.

Hypotheses are again divided into *working* and *descriptive*. Whenever a hypothesis is set up with the idea of establishing it as a cause or a law, we have a **working hypothesis.** The supposition stands meanwhile as probable, subject to revision and liable finally to be rejected or proved.

Suppositions are often made, specially in text-books, to explain (by way of illustration) the nature of phenomena otherwise difficult to understand. Such suppositions are **descriptive hypotheses.** For example, a fluid, say water, kept in a tank or moving in a current, possesses qualities similar to those of electricity. Suppose then that electricity is a fluid, and the facts observed in water will give an idea of the effects of electricity. Descriptive hypotheses, therefore, are not devised in order to be tested and proved, but merely as comparisons to illustrate invisible or obscure facts.

4. The Use and Misuse of Hypotheses. It is natural, as noted before, to make suppositions on every new case to be explained. Leaving out obvious facts, it may be said that hypotheses are the necessary or ordinary means to use in scientific inductions. It is noticeable that as an investigation grows difficult, more and more hypotheses are advanced as possible means to a solution.

A hypothesis is useful in many ways. It brings many facts into unity or under a common relation ; it affords a temporary explanation ; it clears somewhat the ground entangled with complicated relations ; and above all it makes a beginning in the process of proof. A hypothesis, no matter how feeble, and even if finally rejected, may have served a purpose by pointing out another hypothesis as a better way towards the solution of the problem.

Nevertheless hypotheses are misused by excess of confidence, when we rest on them as proofs. A hypothesis is no proof whatever. It merely says that a factor A may be the cause of a fact B. But to argue from possibility to reality is illegitimate ; things actually done are certainly possible, but things possible are not necessarily to be realised.

5. Conditions of a Good Hypothesis. A hypothesis, to begin with, must be **real** ; that is to say, about something that exists in the order of nature. Newton employed the words *Vera causa*, a true cause, not precisely to mean that the supposed cause must be well known in its qualities and from actual experience, but in the sense that its existence in the order of nature is at least inferred from other experiences. This first condition is essential, since natural phenomena are linked together by unity and continuity. To explain nature by unnatural laws or causes is a fiction not allowed in experimental sciences. "*Hypotheses non fingo*" said Newton.—"My hypotheses are not fictitious or gratuitous." Undiscriminating minds are likely to embarrass themselves with such confused hypotheses. All superstitions rest on hypotheses of this sort; as, for instance, when a bird entering the chamber of a rich man is supposed to be a sign of death, or a certain day of the week to be unlucky for one who starts on a journey.

Besides, a hypothesis must be **consistent**; that is to say, consistent with itself, with other standing hypotheses, and particularly with the acknowledged laws of nature. If the consequences that follow from a hypothesis are at variance with each other, it is self-inconsistent; and if they contradict any of the laws of nature, the hypothesis must be abandoned, for there can be no opposition among the various facts that constitute the unity of nature. But a hypothesis may rival another hypothesis on the same matter; and then one of the two must go. A hypothesis need not be rejected at once because of some difficulties; it may be reformed or limited, so as to make it agree with the whole system of science. A striking instance of a difficult and yet standing hypothesis is the supposition that a substance called ether pervades all material spaces and serves as a medium of the transmission of light. For the propagation of light demands qualities in the ether that are difficult to understand.

A third condition is that a hypothesis be **clear and definite**; in other words, capable of verification. If clearly conceived, its consequences will be obvious and explicit ; and if definite, we can apply measurement and express it in mathematical formulas. Newton did so in his theory of gravitation ; and the consequences of the wave-theory of light are obvious from our experience of wave-motions in liquids and in sounding bodies.

Clearness is not the same as simplicity ; yet on equal conditions a simple hypothesis is preferable to a complex one.

6. Verification of a Hypothesis. A hypothesis is a supposition that we make as to the possible cause or explanation of a phenomenon. Observe that previous to any investigation we cannot say whether it is true or false, but we certainly know that our supposition, if true, must account for the conditions existing in reality, or else has to be rejected. Mill's definition (given above) says that a hypothesis is put forward in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real. This is precisely what is meant by the process of verification, the putting of a hypothesis to the test of facts.

We should distinguish between direct and indirect verification. Whenever the facts to be accounted for are under control, and the hypothesis is subject to experience, there is no process of deductions. A student, for instance, who finds himself without the books, may suppose them to be in the clerk's office. The hypothesis can be proved or disproved by mere observation. Again, people working in the fields are accustomed to look upon the moon as being the cause of frequent changes in the weather, a supposition that the moon and the variations in the weather succeed each other. To take another example : Suppose a water supply happens to be suspected of being injurious to health; the chemical analysis of its contents will establish whether the hypothesis is or is not true to fact.

Very often, however, a hypothesis does not come under observation, and we resort to deduction as a means to bring it down to the reality of facts. In this situation one has to reason out what should be expected, if our hypothesis were true, and has to draw practical consequences that follow logically from it. Then each of the consequences is compared with facts of experience. A single truly erroneous consequence overthrows the truth of our hypothesis; for no falsehood follows logically from a true premise. A hypothesis in this plight needs must be modified or rejected. But if all the consequences agree with facts, the hypothesis is said to be established or verified. A hypothesis is extended when it explains more facts than those originally intended; and this is a great test in its favour. A striking illustration is found in gravitation, devised first as a hypothesis to explain the movements of heavenly bodies, and extended later on to account for many other facts, until it has been acknowledged as a universal property of matter. Another typical example is that of the periodical classification of chemical elements introduced first by Mendeleeff. He not only explained the successive atomic weights of the elements then known and many of their actual properties, but led to the discovery of new elements. His generalisation was justified by subsequent investigation and has become a law of nature.

To sum up. Whether the process of verification be direct or indirect, we arrive at the same place by different roads. It always comes about that a hypothesis begins at facts and ends at further observation of facts.

7. Crucial Instance. It may happen that two distinct hypotheses explain, or seem to explain, a given phenomenon. A crucial instance is an experiment chosen to decide which is the true one. The experiment is so devised that if the result is fully in harmony with the one, it totally contradicts the other. Some examples are conspicuous in the history of science. The Newtonian theory of light claimed greater velocity when passing from air to water; the wave-theory maintained the opposite. The experiment decided that the velocity of light is less when passing to a denser medium, say from ether to air, or from air to water.

Again, gases are reduced in volume by compression. Are the intermolecular spaces reduced or is the matter compressed? The experiment was carried out by the spectrograph, where the significant line would be displaced, if the matter were compressed. On trial it was found that the significant line was not displaced; and so the matter was not compressed.

8. Proof of a Hypothesis. To prove a hypothesis is a further and much more difficult step. All methods of induction are ways of proving hypothesis with unequal result, as pointed out in the discussion of those methods given further on.

Establishing a hypothesis means to remain in the expression: "If X then Y." In Mill's definition it is wisely said that we test the hypothesis with real facts "under the idea that, if the conclusions to which the hypothesis leads are known truths, the hypothesis itself must be, or at least is likely to be true." Truth may follow from false premises; wherefore a proof is not accomplished merely by ascertaining the agreement of a hypothesis with facts, until we push forward to demonstrate that it is the only one, namely : " Only if X then Y," which is equivalent to the reciprocal of causation, " All Y is X." This is the ideal of inductive inference, to arrive for certain at a universal proposition as a matter of fact ; in other words, a law of nature. Such an ideal has been obtained in Physics and Chemistry. For instance, an electric current produces work by moving a motor, and the same motor by a similar amount of work will generate a fresh electric current equal to that spent. Again, water is decomposed into hydrogen and oxygen, and the resulting volumes, two of H and one of O, will bring back in return the previous amount of water. In these cases the four axioms of causation are realised, and the cause is fully ascertained.

9. **Degrees of Generalisation.** We have seen that by proving or establishing a hypothesis we arrive at a generalisation. But not all generalisations are of the same value or degree in universality or in certainty. A moral generalisation is nearly universal ; for instance " All boys are playful." A particular or a probable proposition is not universal ; hence we say, " Most metals are solid ; graduates in the universities of England will probably be successful in India." A hypothesis satisfactorily proved is called a law of nature and is meant to be strictly universal, as the following : " Heat expands bodies." There may be a known exception ; as in this law, where water at the freezing point expands.

Exceptions, however, confirm the rule, in the sense that a law, the exceptions of which are definitely known, becomes certain and strictly universal by means of this limitation ;—that is to say, its extension is fixed by the exception.

It is clear that approximate generalisations express the knowledge derived from experience by the process of induction. Nature, outside the mind, is determined precisely the same way in its various relations ; but to this uniformity of nature we only approach by such generalisations.

10. The Laws of Nature. Law in its primitive meaning is a rule of conduct or a command binding individuals to obey under a penalty. The name law has been adopted, by analogy, to signify the fixed relations guiding the continuous motion in nature, or establishing order among the constant changes of the universe. The resemblance is perfect in uniformity, obedience, and also in sanction. The difference lies in the matter ; a moral law rules human actions, while the law of nature directs material events.

Primary laws are the widest uniformities, having no other higher laws above them. Examples : The law of gravitation; the atomic theory; the wave-theory of light.

Secondary laws are less general uniformities, mere applications of higher and simpler laws by combinations of elements or forces. Examples : The laws of the pendulum ; the law of tides ; the rising of water to the height of 33 ft. in a pump ; the laws of reflection, refraction, polarisation of light, etc. Derivative laws are secondary laws clearly deduced from a higher law, by which one explains what they are, and the reason for their existence. The examples given above are laws derived from gravitation or from the wave-theory of light.

Empirical laws, or statistical laws, are uniformities for which we possess no other reason than uncontradicted experience. But as nothing exists without a reason, they must be considered as secondary laws underived for the time being, yet derivable as we advance in knowledge. All exceptions to a general law are empirical laws. Other examples :

Mercury salts are poisonous ;

The atomic weight of an element multiplied by the specific heat is equal to 6·4 ;

Changes of weather correspond to certain appearances in the sky ;

Peculiar laws of ebb and flow in the tides of particular localities ;

Certain animals are naturally enemies to other animals, etc.

Empirical laws are much less certain than derivative laws.

Laws of succession and laws of co-existence. The former consist in one event invariably following another, as in causation. The latter are those by which one thing or property is constantly found side by side with another. Succession may be direct and immediate, or indirect and mediate. Heat directly expands bodies. Evaporation directly reduces temperature. A bad crop brings about a rise in prices mediately or indirectly.

Laws of co-existence are found in great variety. Examples : Certain qualities are constantly found together in individuals of the same kind ; the properties of matter, gravitation, extension, inertia are naturally inseparable ; opposite seasons in opposite hemispheres remain constantly in the same relation ; the musical notes of the diatonic scales are in a fixed natural order ;

and generally speaking all co-effects are examples of co-existence.

The peculiarity of these laws is that they cannot be traced to a higher law of co-existence by derivation; but they may be referred to a common cause.

Invariable laws are such that no exception is ever expected to occur; and if a seeming exception appears, it is tested and found not to be genuine. Laws to which exceptions may arise, are **variable** or uncertain, and held with misgivings; for instance, the assertions: "Cases of cancer are incurable; Comets move around the sun from east to west," are not so certain that exceptions may not happen.

11. Scientific Explanation. To explain means to make plain, to account for something by giving a reason. In ordinary life, any motive that satisfies one's curiosity or mental capacity is an explanation, which, of course, varies with different persons. Many occurrences are explained away by saying: "It is the custom."

Scientific explanation goes deeper into the nature of facts, and consists in referring them to their causes or their laws as discovered by a process of induction. In mathematics, a problem is explained by applying a general formula or rule, and a theorem by showing that it follows from another truth already established. In concrete sciences, likewise, a phenomenon is explained by pointing out the cause. A mere statistical law, an analogy or a comparison that throws some light on the way things come about, are also in some sense an explanation. An explanation is always a generalisation or a broad conception that bears relation to individual facts of the same kind.

The modes of explanation may be reduced to the following :

1. Analysing a compound phenomenon into its separate causes, say **a, b, c,** into **A** = **a, B** = **b, C** = **c.** Thus the parabolic motion of projectiles is explained by the three forces, impulse, gravitation, resistance.—Likewise the perfections or imperfections of a photograph, which is a compound effect, may be traced one by one to the separate factors in the compound cause that brought about the whole effect.

2. Tracing a phenomenon up to its highest cause through a chain or continuity of causes , e.g., saying that **c** is from **A,** because **c** is from **B,** and **B** is from **A.** Thus the rising of the water in a pump to a height of 33 ft. at sea-level is explained by the pressure of the air, and that pressure by gravitation.

3. By grouping various secondary laws under a simpler and more general law, of which they become so many applications ; e.g., several laws, **a, b, c,** are traced to a primary law **A.** Thus the many laws of Multiple Proportions in chemical combinations were reduced by John Dalton to mere consequences of the Atomic theory.

12. Limits of Explanation. Explanation is the outcome of the process of induction ; hence it is limited by the slow process of scientific enquiry, and by the many obstacles in the way of the study of nature. To begin with, we can never fully grasp all the attributes and relations of individual objects, so as to comprehend them. Take any common stone. By analysis we may know the chemical elements, the properties of weight and temperature, perhaps its geological origin ; but why it has that particular shape, its antiquity, its infinite relations are beyond our reach.

Besides, the various activities of life, such as sensations of light and sound, and facts of consciousness like pleasure and pain, are not known sufficiently to be reducible to one common mode of action.

Empirical laws rest on statistics, or numbers of facts, and await explanation. Likewise many hypotheses, such as that of the evolution of species in Botany and Zoology, fall far short of explaining the gulf that separates the inorganic from the organic kingdom.

A great number of secondary laws are successfully explained by showing their connection with a higher law ; yet a further enquiry into the cause of primary laws is not possible from experience ; and we rest on the supposition that there is a redistribution of matter and energy. Hence on all sides we are bound to arrive at something unexplained, and have to rest satisfied with the bare fact.

Nevertheless, all processes of induction lead to generalisation for the purpose of knowledge. Classification itself, though hardly an explanation, generalises the characteristics discovered in things by means of a concept or common name ; and many kinds of names are united in a series.

13. **A System of Knowledge** is the orderly arrangement of all the known truths on a special subject-matter into a connected body of doctrine, showing how one is dependent on another. A system of knowledge in experimental sciences is the outcome of induction and explanation. The more science advances, the better is our knowledge systematised. The ideal is to co-ordinate all the sciences into one system, in such a way that the whole structure of human knowledge be made to correspond to the marvellous unity that holds together all the activities of the universe. To this goal scientists may successfully approach by further discoveries.

Meanwhile it is given us to see the harmony among the various parts of a limited science. It is said that deduction begins where induction ends; and this is quite true in experimental sciences—that is to say, we are able to prove by deductive inference as much as has been generalised from facts. All synthetical propositions, once established, are so many premises from which we can argue to particular cases. True general statements may be held either by authority or by personal inductive evidence; but this makes no difference for our purpose. Our premises will be based on reality, and our conclusion proved because contained in those premises. Thus it comes about that by induction we complete the circle of our reasoning, upwards and downwards, from facts to mind and from mind to facts—which is the problem proposed for solution at the beginning of induction.[1]

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[1] Herschel in his admirable *Discourse on the Study of Natural Philosophy*, Chap. II., No. 13, says: "Science is the knowledge of many, orderly and methodically digested and arranged, so as to become attainable by one. The knowledge of reasons and their conclusions constitutes *abstract*, that of causes and their effects, and of the laws of nature, *natural science*."

CHAPTER XIII

DIRECT METHODS OF INDUCTION

1. Scope of These Methods. We come to explain the final stage of inductive process, that is, the verification of hypothesis. The problem before us is no other than what we proposed at the beginning of induction, namely, how to discover the scientific cause of a given effect, or, generally speaking, how to arrive from facts to a general proposition which will be the expression of a natural connection between subject and predicate. We have seen that facts properly observed and interpreted give rise to a reliable hypothesis that suggests where to look for a possible cause or relation. Our purpose is now to develop the manner of testing the hypothesis. The various ways in which we carry out this verification are called Methods of Induction.

Regarding hypotheses we made a distinction between direct and indirect verification ; accordingly there are two kinds of methods, those in which the hypothesis itself is being observed, and those where the hypothesis is brought down to facts by a roundabout way. Both kinds must be considered separately.

We begin with the direct methods which are ways adopted to establish precisely the cause of an effect or the effect of a cause by the aid of observation and experiment. These methods, according to John Stuart Mill, are as follows :

The Method of Agreement.
The Method of Difference.
The Joint Method of Agreement and Difference.
The Method of Concomitant Variations.
The Method of Residues.[1]

2. The Basis of These Methods. Each method begins by instances of the phenomenon under investigation, that is, by repeating the phenomenon, not for the sake of number, but to compare those instances with one another. Suppose, for example, that a mango-crop fails ; the problem is to ascertain the cause which brought about the bad crop. Previous experience may suggest as a probable cause an early storm. This is your hypothesis and may or may not be true. Proceed to take notice of several instances of the same phenomenon in similar circumstances, that is, compare several mango-crops and see how they succeeded with and without an early rain. Suppose again, a number of patients are suffering from fever, the evidence of which suggests that in all cases of them it is malaria. A simple analysis of their blood may reveal a difference among them, thus calling for another hypothesis. Take another illustration. The phenomenon to be accounted for is my illness after dining out. Considering the case, it occurs to me that a particular food may be the cause. The method of inquiry will be to compare several meals in which I always eat that particular food.

These examples go to show what is meant by instances of the same phenomenon. The instances are not exactly the same; their conditions are somewhat different. In fact they are intentionally so arranged that, if compared with one another, there are instances in which the effect follows, and other instances in which the effect does not follow. Going back to a previous example, some mango-crops have failed while others have not, in other words, we have positive and negative instances of the same phenomenon. Or supposing that the instances are all positive, yet there will be variety among the conditions. In each comparison we watch carefully the elements which can be absent in the presence of the effect, and those which can be present in the absence of the effect.

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[1] Mill, *A System of Logic*, Vol. 1., pp. 425-81.

What are we aiming at in these comparisons? The idea is to focus down upon that which appears to be relevant and necessary for the production of the effect, by separating it mentally from anything else which is merely accidental and irrelevant. In other words, we perform elimination, relying on very simple principles derived from the characteristics of causation explained before.

(*a*) Whatever is absent in the presence of the phenomenon is not the cause: the axiom of the Method of Agreement.

(*b*) Whatever is present in the absence of the phenomenon is not the cause: the axiom of the Method of Difference.

(*c*) Whatever does not vary is not the cause of a phenomenon that varies: the axiom of the Method of Concomitant Variation.

(*d*) Whatever is the adequate cause of a phenomenon is not the cause of a different phenomenon: the axiom of the Method of Residues.

Although these axioms are fulfilled in the various methods, yet they are only negative statements, telling us what is not the cause. The causal relation does not come under our senses, it must be inferred. In other words, the methods pave the way for the mind to discover the cause.

3. The Inductive Inference. An inductive method may be used in ordinary life as well as in science. But in any case we must rely on the general principle that every event must have a cause, for without it we could not pass from sequence to consequence. That is to say, from the fact observed that one thing follows another, we could not argue to the conclusion that one event depends upon another. Scientific induction, however, goes further, for it aims at discovering that set of antecedent conditions which are necessary, neither more nor less, to produce the effect. When this conception of the scientific cause is realised in the methods, we are in full possession of all the axioms of causation, that is, we can argue from cause to effect and from effect to cause. Moreover by another premise, the uniformity of nature, we are able to generalise from what is true in one case to all similar cases. The final conclusion is a law of nature, which is the goal of scientific induction.

At this point the question arises : is the universal conclusion true to fact ? Or is it certain ? The answer is very simple ; if the premises are true, the conclusion must be true. Are then the premises true ? The principle of causation and its implications are presupposed in every process of induction. What makes the difference is the meaning one takes of these grounds of induction. If causation and the uniformity of nature are understood in a real sense, that is, as included in the order of nature, there is no more to say on this account. An inductive argument yields a conclusion in accordance with facts and imparts true knowledge. Any weakness in the conclusion must come from the premises that rest on the analysis of facts. Hence the certainty of the conclusion, or its amount of probability, depends on the method alone. How far each method can attain to the precise scientific cause is the whole difficulty met with in solving the problem of induction. Now each method must be considered separately as to the process and the strength of the argument.

4. **Method of Agreement.** Mill's canon is this: "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause of the phenomenon." Accordingly we compare several instances of the same phenomenon, all positive instances, in which both the suggested cause and the effect are present. By supposition, therefore, the instances agree in having some antecedent conditions always present when the effect is present. Wherefore the process is called the method of agreement.

Take as an illustration the pendulum. It was for a time a hypothesis that the time-oscillation is related to the length. The hypothesis may be verified by the Method of Agreement with several pendulums so arranged that all having the same length differ in weight, shape, material substance. Comparing them with one another it will be observed that the time-oscillation is practically the same in all of them. We conclude, therefore, that the conditions of weight, shape and material substance are indifferent to the time-oscillation and can be eliminated as irrelevant to the effect. The connection between length and time-oscillation is not seen, but can be inferred by inductive reasoning as follows :

Whatever can be absent in the presence of the phenomenon under investigation is not the cause. But the conditions of weight, shape and material substance can be absent in the presence of a fixed time-oscillation, therefore those conditions are not the cause. But there must be a cause, according to the principle of causation. Therefore length, the condition present in the presence of the fixed time-oscillation, must be the cause, or part of the cause. But the same cause always produces the same effect, according to the uniformity of nature. Therefore any pendulum of that length is bound to have the same time-oscillation.

The method is also applied to groups of instances, when simple instances are not enough to ensure elimination. For example : A substance is supposed to be a preventive of cholera. Let it be applied by inoculation to all the members of five families in a locality when the epidemic is raging high. If all the members of the five families remain immune, there is a reason to believe that the substance in question had influence in the result of preventing disease ; for neither the age, nor the conditions of health, nor the habit of occupation and so forth, different among the members, nor again the place or some sort of hereditary condition different among the families, have anything to do with the result of counteracting the epidemic.

For the sake of simplicity the method may be expressed symbolically. Break up the phenomenon mentally into supposed antecedents and consequents, and represent the former by capital letters and the latter by small letters, marking the suspected cause by **X** and the effect by **y**.

Instance (1) **A B X—h m y**; instance (2) **B C X—n q y**; instance (3) **A C X—r s y**; then **X—y** are likely to be connected. Observe the following: **A** is absent in instance (2), **B** is absent in instance (3), and **C** is absent in instance (1) in the presence of the effect **y**, wherefore all can be eliminated, and there remains only **X** as an invariable antecedent condition of **y**.

As to the logical value of this method, we can say that the instances employed being positive and so to speak ready made for us, are easily available and can be compared by mere observation. On this score the Method of Agreement is often used in occurrences of daily life to ascertain roughly a cause, and the result amounts to sound probability. Moreover, even in science the usefulness of this method as a means to direct us towards discovery cannot be doubted. But with observation alone the method is not conclusive, at least is not so in discovering the precise scientific cause. It will not ascertain what is optimistically stated by Mill, "the circumstances in which alone all the instances agree."

This method is essentially imperfect in many ways. For, in the first place, very often we get an invariable antecedent, but that thing is not the cause. Thus, for instance, a switch-on is an invariable antecedent of light and is not the cause. Secondly, an invariable antecedent may not be the precise cause, as in the case of polluted water that brings about disease. It is not the whole of water, but certain germs in the water that are injurious to health. Similarly swamps are taken to be malarious, when in reality they are only a breathing place of those mosquitoes which cause malaria.

Thirdly, two things may follow each other and, if compared by this method, may seem to indicate causality, when in fact they are co-effects from another cause, as happens in the seasons of the year. Fourthly, not unfrequently people allow their minds to be swayed by feeling and imagine they see a phenomenon where it is not. The way out of the difficulty is to reverse the method producing the phenomenon by experiment.

Take an instance in point,[1] a recent investigation on scientific lines. Indian plant remedies are said to cure a snake bite, because on their application the snake-bitten man gets better. This, however, proves nothing as to the efficacy of those plants. The beneficial results apparently observed are unreal. Everyone who is bitten by a snake expects a rapid death, but in fact only a small proportion of bites are dangerous. Hence every variety of a snake-bite cure has, for a time at least, acquired a reputation.

The scientist wants to be absolutely sure that the snake-bitten man was indeed in such a condition that death would have followed if the said remedy had not been applied. In other words, the scientist wants to make sure that the victim has absorbed in his system a lethal dose of poison. This is, however, far from being the case.

In order to be actually poisonous and deadly, a snake must possess a tolerable amount of effective poison, and be supplied with a proper instrument for injecting it ; if either of these two conditions fail, the snake will have to be classified as practically harmless to man. Of some forty or so species of snakes commonly met with in the plains of India six only are poisonous, and in many instances a highly venomous snake fails to inject its victim with a lethal dose of venom, either because the snake was not in a fit condition, or else the person bitten was partially protected.

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[1] Extract from a paper on " Indian Plant Remedies used in Snake Bite," Dr. K. S. Mhaskar and Rev. Father T. F. Caius, S.J.—Pharmacological Laboratory, Haffkine Institute, Bombay, 1930.

There is but one way of testing these remedies. The lethal dose of poison has to be artificially administered to a healthy animal, the remedy whose efficacy is either asserted or denied has to be immediately administered, and the results have to be carefully watched and recorded. By repeating this process a number of times on different animals, strictly scientific results are surely obtained. Now this is precisely the method which the authors have followed. Their final statements are these : " In all we have tested 314 individual plants and 184 combinations. We have every reason to believe that our work is exhaustive. We may safely conclude that none of the Indian plants recommended for the treatment of snake-bite has any preventive, antidotal or therapeutic effect."

5. Method of Difference. The canon according to Mill is this : " If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur have every circumstance in common save one, that one occurring only in the former, the circumstances in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." We suppose two instances perfectly equal and, if possible, identical in all respects, except for one element which is simply present in the one and absent in the other, but this element is such that when present the effect follows, and when absent the effect does not follow. This method is meant to prove that a certain element is the only factor responsible for the production of the effect. Two instances alone are employed to ensure perfect knowledge of what is in the instances and consequently perfect elimination. Two instances as described are not obtained by observation of things in the course of nature, they must be made artificially. Hence this method is one of experiment. Furthermore this method is particularly suitable to ascertain either the cause or the effect; it all depends upon what is known about the conditions of a phenomenon. If an antidotal serum is chemically known, we try the effect under varying circumstances, and vice versa, if the disease is known we attempt to find its antidotal drug.

A familiar example will illustrate the process. Take two glass vessels; fill either one with ice-cold water, and the other with the same liquid at the temperature of the air, and watch what happens on the outer surface of the vessels. The surface of the latter glass vessel remains dry, while that of the former becomes gradually covered with condensation of moisture. All circumstances in the experiment being the same in both instances, but for the presence of cold where the effect follows and its absence where the effect does not follow, we conclude that all circumstances other than cold have no influence on the effect under investigation.

Take another example. The production of sound was proved to be due to the air, by two identical instances consisting in a sounding bell enclosed in a vessel with and without air. In the total absence of air no sound could be heard, but on the air being re-introduced the sound was heard.

The symbolical expression of the method is this :

First instance : **A B X—a b y** ; second instance : **A B—a b** ; hence **X** is causally connected with **y**.

The reasoning runs as follows : What can be present in the absence of the phenomenon is not the cause ; but **A B** are present in the absence of **y**, therefore they are not the cause. But there must be a cause, therefore **X**, the only condition present when **y** is present is the cause. But the relation between cause and effect is uniform, therefore **X** is always the cause of **y**.

The method of difference is highly efficient, but requires experiment. Nature does not give us two instances of the same phenomenon that differ only in one thing. Whenever this method can be worked out to full satisfaction, it proves by itself the existence of causation, and in that regard is by far the best of all inductive methods. The precise scientific cause, however, may not be distinctly set down by the most exact experimental technique. What is called the only antecedent condition is often combined with something else completing the cause. In the given example of sound, for instance, air is not the whole cause ; air is the material element which together with the wave-motion produces the sound. Nevertheless the example proves a natural connection. This difficulty and similar ones that can be put concerning the precise scientific cause, are not so much defects in the method as difficulties unavoidable in the study of nature. In other cases in which we compare instances of the same phenomenon by mere observation, the process of this method will not prove, generally speaking, a natural connection, and then we fall back into the objections raised against the Method of Agreement, namely, the antecedent condition present and absent may be the common cause, a symptom, or no cause at all.

6. The Joint Method of Agreement and Difference. Mill formulates the canon thus : " If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause of the phenomenon." The rule supposes two sets of instances ; one in which the instances are positive, another in which the instances are negative. Both positive and negative instances are of the same phenomenon, but differ among themselves in possessing various conditions. There is one antecedent that happens to be present in all the positive instances where the effect follows, and that same antecedent is absent in the negative instances where the effect does not follow. The two sets may be compared by mere observation when found in the course of nature. Thus it has been observed that a number of guests in a banquet got ill after taking a certain dish, while another set in the same banquet, who did not partake of that dish, felt comfortable with no signs whatever of any illness. The method works well in preference to any other method, when the phenomenon is such that great numbers are involved, and yet analysis or elimination is difficult.

Take the following illustration. Doctors in charge of sanitation acquire evidence as to the efficacy of vaccination by this method. An epidemic breaks out ; then a certain form of vaccination is applied to various persons who differ in age, mode of life, etc. If those vaccinated are not attacked, and those not vaccinated are attacked, there is evidence that the supposed remedy is effective. The symbolic expression may be this :

Positive instance (1) **A B X—h m y** ; positive instance (2) **C D X—n g y**. Negative instance (1) **A B—h m** ; negative instance (2) **C D—n g**. The positive instances agree in that **X** is followed by **y**, and the negative instances agree in that **X** being absent, **y** does not follow. The two sets differ therefore in both **X** and **y** being present or absent.

The reasoning runs this way : Whatever can be absent in the presence of the effect, and can be present in the absence of the same effect, is not the cause. But the antecedents **A B C D** comply with both statements, therefore they are not the cause. But there must be a cause, therefore **X**, the only antecedent present in the presence of **y**, and absent in the absence of **y**, is the cause. But the same cause always produces the same effect, therefore **X** is always the cause of **y**.

This method will surely fetch a far better result than the Method of Agreement alone, no matter how many instances are involved. The elimination made by positive instances is confirmed by negative instances of the same phenomenon. It resembles the Method of Difference in that a certain antecedent condition in the positive instances is absent in the negative instances, but instead of two identical phenomena we employ here two sets of instances of the same phenomenon. However, the conclusion drawn from the Joint Method is not likely to be certain, when intended to compare naturally occurring phenomena of which no two are equal. As the variety of circumstances grows with the number of instances, there may be among them something unknown and hence elimination is hazardous. We may not be absolutely sure that only one antecedent condition is present in one set and absent in the other. But the method carried out by experiment with full control of the prevailing conditions, can yield a certain conclusion.

7. The Method of Concomitant Variations. Mill's canon says: "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation." The process consists in comparing two phenomena such that while one varies the other also varies, and we suppose variations which occur with certain uniformity, no matter what the form of variation may be. What the method requires, is constancy and uniformity in the two phenomena that vary together. Thus the temperature and the volume of a gas vary directly; but the pressure and the volume vary inversely, and gravitation varies inversely as the square of the distance. Variations of the form described are a source of discovery. Take the following example: The phenomenon of the tides has been traced to the attraction of the moon and the sun, because the flow and the ebb of the ocean are in correspondence with the successive positions of those heavenly bodies. Moreover, knowing the variations of the cause, namely, the force of attraction, we are able to predict the variations that must follow in the effect. The time-tables of the tides in different localities are the result of this calculation.

The symbolical expression of Concomitant Variations is like this: (1) $\mathbf{A\ B\ X_1}$—$\mathbf{a\ b\ y_1}$, (2) $\mathbf{A\ B\ X_2}$—$\mathbf{a\ b\ y_2}$,

(3) **A B** X_3—**a** b y_3. This represents a constant variation where **X** and **y** are the varying phenomena. The reasoning on this method is as follows: Whatever is constant is not the cause of a thing that varies, therefore **A B** constant are not the cause of **y** that varies. But there must be a cause, and besides cause and effect are equal; therefore **X** which varies together with **y**, is related to it by some kind of causation.

The method is deficient in two points; first, it lacks the power to point out causes, because two or more phenomena can vary together as co-effects of a common cause. Two tram-cars, for instance, vary their movements together, and yet one is not the cause of the other, but both depend on the same cause, the electric current. Secondly, a relation between two sets of variations may be constant and uniform, and yet the method is not enough to show which must be the cause and which the effect.

On the other hand, the Method of Concomitant Variations is most useful and unique in the study of complex phenomena, when elimination is impossible by the other methods. For instance, changes of weather, magnetic storms, sun-spots, earthquakes, and the like are too big, or too far away to be analysed by comparing several instances Moreover, there are conditions in nature, such as gravitation, heat, atmospheric pressure, friction, human character and so forth that never cease in their action and so cannot be eliminated, but, as these things vary, we can study them in their degree.

Variations, again, afford the means of measuring the cause by the quantity of effect and vice versa. This characteristic is the source of quantitative inductions of which the thermometer is a neat example. This familiar instrument measures the cause (heat) by its effect (the expansion of a column of mercury) ; the degrees marked on the thermometer are degrees of heat in the surrounding atmosphere. Modern science endeavours to establish quantitative relations like that between heat and expansion in every branch of inquiry. Two phenomena related to each other are precisely measured according to a standard unit, and the results are usually expressed in mathematical equations, or chemical formulas. Such quantitative inductions are general laws embracing practical cases under similar conditions. This application of the method accounts for the modern tendency among logicians to think of relations, forgetting the cause. We should remember, however, that a relation cannot subsist without the terms related, which are no other than cause and effect, or co-effects of the same cause.

Concomitant Variations are nicely put together before the eye by graphic representations of various forms. If, for example, a horizontal line marks the time, and a vertical line measures the amount observed, the resulting curve will give us the constant variation. Thus, a single diagram can show at a glance how the elements of temperature, humidity, cloudiness, evaporation and irradiation have varied together during a day, a week, or a month.

8. The Method of Residues. Mill's canon is as follows : " Sub-duct from any phenomenon such part of it as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the result of the remaining antecedents." We suppose that a phenomenon has been analysed by a previous method and that a relation has been discovered between an adequate cause and its effect, but there remains something unaccounted for in the latter. For example: Having analysed the atmospheric air into its components, it was found that nitrogen from the air was slightly heavier than that from other sources. The residue led to the discovery of a new gas, Argon. As a further illustration we may remember how the orbits of the planets had been explained by Newton according to gravitation, but there remained perturbations in the movement of Uranus. This residue was accounted for by the existence of another unknown planet, Neptune. To quote yet another example of modern times; the eclipses of the moon are in effect well related to its cause, but a difference has been found between the time calculated and the time observed. This residue must be due to another antecedent which is at present under investigation. Symbolical expression: The phenomenon before investigation is like this: **A B X—a b y**; the result of investigation by a previous method may be expressed this way: **A B—a b**; hence the residue **y** left over must be related to some other antecedent, namely **X**. The reasoning is as follows: whatever is the adequate cause of one thing is not the cause of a different thing; but it has been found that **A B** are the adequate cause of **a b**, therefore they are not the cause of **y**. But there must be a cause of **y**; hence the remaining effect must be related to some other antecedent.

At first sight the process resembles the Method of Difference, and as a matter of fact the residue is a difference in the effect, but such difference is not the result intended by the previous method; on the contrary, it is an exception that leads to further inquiry. Properly speaking the Method of Residues does not discover anything, but suggests a hypothesis and opens the way to discover the unknown cause. The application of this method is quite frequent in our days, because in proportion as new discoveries are brought to light, more and more are the exceptions, either by excess or by defect, which await explanation.

9. Criticism of the above Methods. Whewell objected that Mill's inductive methods take for granted the very thing which is most difficult to discover, the reduction of phenomena to formulas. In the presence of a complex fact, where are we to look for the combinations **A B C**? Nature is not made according to these combinations. Besides, the methods overlook the counteracting conditions. To put it bluntly, the above methods are of no use to discover order, causation or law in the course of nature.

To this it may be answered that Mill's Methods of Induction are not meant to represent how nature is actually constituted, but only lay down rules to be tried in the interpretation of facts. They are not infallible nor universal; on the contrary, they bring about in practice unequal results. Nay more, many complete phenomena defy these methods altogether. Nevertheless, no one can deny that the proper way to approach the study of nature is by simplification, that is, clearing the field of inactive or unrelated conditions or elements. And what else are the principles of agreement and difference intended for? Discrimination and insight are qualities of the mind required together with precision in the practice of the methods, but they are not the method itself. The real objection to our methods is the supposition that an antecedent **A** with

B and **C** is related to a consequent **a** with **b** and **c**. We must consider that elements in nature cross one another in many ways, which are subtle and unknown to us. On this account no method can adjust itself precisely to the conditions of natural phenomena.

A further question is whether Mill's canons should be considered as methods of discovery or rather as methods of proof. They are both, but primarily they should be taken as methods of proof, that is to say, as ways and models to exhibit the reasoning involved in scientific process. These methods are not intended to be used in the form and in the order proposed ; the scientist is free to follow his own ways as the circumstances demand. It must be remembered that there is no royal road in research. Every step has to be taken more or less at random, for the investigator is in an unknown country still unmapped. What we say is that those scientific ways of inductive inquiry can be reduced for the purpose of Logic to one or other of Mill's ideal forms.

Again, exception has been taken to the name " inductive methods " ; according to some critics they are not inductive but deductive. The answer is that any inference made by these methods is at once inductive and deductive ; without deduction we cannot arrive at the existence of a cause, as pointed out before. One of the premises must necessarily be an axiom of causation combined with the inductive premise. Furthermore, the ultimate premise by which we generalise is the uniformity of nature. But for all that, Mill's methods are certainly inductive and mainly so, because the difficult point that makes inference probable or certain lies in the analysis of facts, consequently the conclusion depends upon the inductive premise.

A review of these methods as a whole makes clear a distinction between them. The three Methods, Agreement, Difference and Joint Method are said to be qualitative, because by them we discover the kind of relation that exists between two phenomena. The other two methods, Variations and Residues, are called quantitative, because they are means to establish how much of the cause corresponds to the effect. On the other hand, there is unity in these methods, because all may be reduced to a point of difference, that is to say, the purpose in each method is to realise a difference between relevant and irrelevant conditions. Thus, in agreement some conditions are absent when the effect is present, while other conditions are constantly present when the effect is present, and this is a point of difference. In the Method of Difference it is also plain that certain conditions can be absent while others cannot in the presence of the effect. In Variations some conditions are not varying while other conditions vary, which is again a matter of difference. In Residues the greater part of a phenomenon is connected with the cause while a remainder is disconnected, and this again is a point of difference. What makes to our purpose is that the strength of the methods consists precisely in realising a point of difference for the purpose of elimination. Hence although the process is not one and the same, all may be reduced to one fundamental principle.

CHAPTER XIV

INDIRECT INDUCTIVE METHODS

1. Inductive Deductive Methods. We propose taking up the direct verification of hypotheses, a process in which deduction and induction lend each other support. The ways of induction already described solve to a great extent a particular problem, namely the discovery of a causal relation. And the analysis consists in comparing antecedents and consequents among which the hypothesis can be traced. This process, however, cannot be applied to all kinds of phenomena. For (*a*) not unfrequently cause and effect are far apart, or at any rate the cause is not a fact that comes immediately under observation. Hence the necessity of testing the suspected cause by deductive inference. And (*b*) the problem of establishing a causal relation is *not all* that induction is called upon to do. There are relations other than causal relations in the order of nature, as when a law is related to another law, or the parts of a system are shown to be united together, or the laws of one system are connected with those of another. These are problems which must be solved by a deductive inductive method, and as the problems are different there are also several ways of employing deduction to bring about unity and subordination in science.

The drift of deductive inductive methods in general is this: We conceive a hypothesis which is likely to explain the phenomenon under investigation. Assuming then that our hypothesis is true, we think out the consequences that follow logically from it and bear relation to another law, or to the facts presented to our experience. This kind of deduction is only intended to bring the hypothesis down to reality and to make possible its verification. The second part of the method is an empirical induction in order to see that our deductions tally with facts. If all that goes well, we conclude that the consequences being true, the hypothesis *may* be true; our expectation grows in probability with the number of deductions which are found to be in harmony with facts of observation. Thus far the process is not yet a definite proof, until by further inquiry we can be sure that our hypothesis is the only one that fits in the whole order or system of facts. When this has been proved, the hypothesis is said to be a fact, or else an established law of nature. Moreover, we should remember that the process will not be conclusive unless both deduction and induction have been logically carried out. Wherefore it is essential that no consequences be concealed out of deference to facts, nor on the other hand, observations be distorted to the end of holding a hypothesis.

2. The Physical Method. Deduction as an aid to induction is not a method exclusively of physical matters, as the name seems to connote; on the contrary, it can be applied equally well to other sciences. A few illustrations will make clear how the method is used in a variety of ways.

In the first place, a law can be strengthened by reference to another law. Examples: The rising of water in a pump was assumed to depend on the atmospheric pressure ; Galileo observed a height of 32 feet ; then by deductive reasoning, his pupil Torricelli calculated that the weight of the air should support a column of mercury 2½ ft., mercury being 13½ times heavier than water. The conclusion was found true by observation, and the previous supposition remained verified. Soon after, the height of the column was seen to vary with altitude above the sea-level. Hence it was concluded with certainty that the rising of water was due to the atmospheric pressure.

Secondly, various laws are related in a system of nature under a wider law embracing them all. Example : John Dalton analysed the chemical composition of various substances and discovered the empirical laws of definite proportions. Considering these results, it occurred to him that every substance may consist of particles possessing a definite weight of their own. From this supposition he deduced that simple substances should have the power of interaction according to certain combining weights. Subsequent experiments largely confirmed his deductions, and the hypothesis became for the time being a very reliable Atomic Theory. But now it is held as true on account of further investigations recently carried out in Molecular Physics. To give another illustration, the Hypothesis of Evolution is an attempt at solving a problem by deduction and induction. The consequences, however, which follow from it are not truly verified, many facts to be explained have aroused considerable discussion ; and the hypothesis is variously maintained by different scientists.

Thirdly, the same method plays a great part in ascertaining causes which are beyond our control. For instance, one may examine carefully a crime, a shipwreck, an explosion, an anonymous document, the progress or decline of an institution, and the like, and yet one may not be able to trace the cause among the prevailing conditions, and has to resort to try the cause by deduction. Example : Let the question be to discover the author of an anonymous document that we have before us. The evidence may suggest one Mr. Smith as the most probable writer. Hence I proceed to deduce what I should expect the author to be, namely, that he had been living at a certain time and place, his acquaintances, his knowledge, his education, his handwriting, and so forth. These consequences bear relation to the document and need only to be verified. I must see that each one tallies with facts concerning Mr. Smith. If so, my hypothesis is highly probable ; it only remains to make sure that no other person can be found in whom those conditions are fulfilled. A similar process is carried out in judicial courts in order to reconstruct a case as it actually took place. In like manner we proceed to inquire into the cause which brought about a remarkable change in the conduct of an individual, or to explain traditional customs, and so on. We should observe, however, that in matters that involve moral considerations, it is often most difficult to obtain a final proof. It becomes almost impossible to ascertain with perfect evidence that our hypothesis is the only one which can account for the facts.

Moreover, there are social events of economic or political character so vast and difficult to analyse that the method just described needs modification.

3. The Historical Method. An attempt at tracing the cause of historical or social events will encounter great difficulty at the very start, namely, in building a suitable hypothesis. For instance, how are we to deal with the fall of the Roman Empire, the success of a battle, the steadiness or unsteadiness of market prices, a general strike, the changes in capital punishment, and a thousand other phenomena of that type ? No direct deductive method is applicable, either because the facts cannot be approached, or because the factors are too indefinite. The evidence from them is likely to be obscure and insufficient to suggest a reasonable hypothesis. The only way open is an empirical investigation leading to a hypothesis which, by the by, will not be a very simple one. The process, therefore, consists in forming empirical generalisations from the observation of other similar facts, known from personal experience or recorded in history. In the case of a battle, see what has actually happened in other battles. If a strike is to be accounted for, study carefully the circumstances of similar strikes in the past and in various places. Thus from previous experience we may draw one or more generalisations that explain past instances. Now assuming that such generalisations are true, we attempt by deduction to bring our case under those generalisations. And this is done, as in the former method, by comparing the consequences that follow from those generalisations with the conditions of the present fact. The process, therefore, involves three stages, induction to arrive at a hypothesis, and then deduction and finally induction or verification. Example : If I wish to find out the cause of the general strike in the cotton mills of Bombay, I shall begin by a study of other similar strikes here or elsewhere. Let us suppose, that on the evidence of facts, I discover a strong propaganda made in each against the owners, both in writing and fiery speeches. Again, that in past cases the living conditions of the workmen had been unsatisfactory in point of sanitation and food provisions. From these two generalisations which explain, to a certain extent, the past strikes under my notice, I shall try to explain the present case. I have to consider the consequences that logically follow from each of those general rules, practical in character and as many as possible. Finally I must test those conclusions, comparing them with all the circumstances of the present strike. If most of them agree with facts, I have discovered the cause or part of the cause with a good degree of probability.

It is most important that our deductions should be strictly verified by facts. Without this, the results obtained by politicians, economists and historians are of little value.

4. **The so-called Mathematical Induction.** The usual way of looking at mathematical reasoning is that, given an example, say a geometrical figure, we draw by a strict demonstration a conclusion. Examining, for instance, a triangle, it is found that the sum of the three angles is equal to two right angles. Then we conclude that the same must be true for all figures which comply with the established conditions. This kind of inference was called by Mill " Induction by parity of reasoning ", because it bears resemblance to the process of real and proper induction. It is interesting to notice how far the analogy goes, and to mark down wherein mathematical inference is not inductive. The process is, no doubt, similar to that of scientific induction. In both there are two questions, the first is to arrive at a conclusion, the

second is to generalise this conclusion. On reflection, however, we can see a remarkable difference. Is the case a particular one which we observe in geometrical reasoning ? It is not. What we really examine is an ideal case, a conception, and not a particular figure. The triangle before us is not this or that triangle, but one in general, made according to previous definitions of straight lines without breadth, meeting each other in a point without dimensions. A figure constructed under such conditions exists only in the mind. Therefore the demonstration comes under the universal, the process being a deduction from definitions, and the conclusion, without further ado, is already universal. This kind of reasoning is not induction but explanation from previous knowledge, just as in the syllogism we draw a conclusion from the premises.

The case is different when we try to discover a theorem or a definition. Original work that begins at tentative analysis is inductive in Mathematics as well as in material sciences. When the mathematician, for instance, comes to consider some members of a series to discover a law, he goes by induction, that is, from particulars to something general, which enables him to supply the missing links in the whole series. And the theory of limits is nothing else but a generalisation based on what is proved to be true in a number of instances. The difference, if any, between mathematical research and scientific induction is that in the latter the conditions of facts are obscure in the complexity of nature, while in Mathematics we can deal with elements which are definite and under control.

5. The Statistical Method. Statistics is a branch of knowledge concerning the collection and arrangement of facts, social or material, with the purpose of finding among them some sort of relation. Thus collecting a number of deaths, we get the relative mortality; counting motor-accidents we can obtain the amount of risk in motor riding; or again, the records of criminal prosecutions suggest the usual criminality of the country. Facts of this kind are very numerous, but taken individually, it is most difficult or impossible to analyse them by any scientific method. They are exceedingly complicated by a variety of circumstances.

The idea of a statistical method is to take them up collectively or in a mass, and to fix our attention upon those attributes which are common or frequent among them, or upon those qualities which often occur together with other qualities. At any rate we rely on an accurate enumeration of instances. From positive instances which are different in degree we can derive the average quality; on the other hand, positive and negative instances yield a percentage. In either form the result is a probable connection, which may be taken as a rule for practical purposes in the normal course of events, that is to say, as long as general conditions remain the same. How this empirical method works, may be illustrated by simple examples. By tossing a coin a few times we cannot experience any relation, but a thousand throws will make it clear that the number of heads turning up tends to equal the number of tails. Thus we get an empirical knowledge of the relation that exists between the two alternatives. Similarly, if a bag contains balls black and white in any proportion, this proportion can be found by experience, to any desired approximation, with the process of drawing and putting back. The remarkable point is that the thread of relation, if any, has to be detected in the long run and not after a limited number of instances. This method, brought into comparison with the other methods described above, is certainly weaker ; it is very rare that a law of nature is discovered by mere statistics. Nonetheless this process is extensively used in Sociology, Biology, Meteorology, and also nowadays in Physics and Chemistry. All Insurance Companies are based on this empirical method, and experience testifies to its practical value. The various ways of calculating on statistical facts and other considerations are not to our purpose and may be studied in the standard books. It is worth noticing that the statistical method is mathematical in form and empirical in fact.

6. Classification of the Various Inductions. We are now in a position to state a distinction between the various kinds of inferences that receive the name of induction. The most essential feature, common to all kinds of inductive inferences, is the process of generalising or ascending from individual and concrete instances to a broad statement, large in extension and narrow in intension. Now under this general conception of inductive inference, Jevons sets a separation between perfect and imperfect inductions, looking at the number of instances, whether all or some only are observed. Hence the best scientific induction is imperfect. His reason is that complete enumeration alone makes an induction certain.[1]

This division does not seem to be wise. A much better distinction is found by inspecting the process, namely, whether facts are deeply analysed or not. If analysis is made to discover in facts the reason or universality, we get induction properly speaking, real and scientific, leading to knowledge of what the thing is. If no analysis is made or no ground discovered to support the generalisation, then our induction is improperly such. According to this fundamental basis, we arrive at the following classification of kinds of induction :

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[1] Jevons, *The Principles of Science*, pp. 146-52.

Proper Induction : An inference based on two distinct grounds : the one material, the facts of experience analysed ; the other formal, namely, a presupposition. Under this head may be placed all inferences by Mill's inductive methods, the physical method, the historical method and mathematical induction.

Improper Induction : A generalisation from facts alone or principles alone. This embraces the following : simple enumeration, colligation of facts, analogical reasoning, percentages and all statistical generalisations, classification, and inference from fact to fact.

Observe that both the proper and the improper induction may be more or less perfect ; and the perfection lies in the degree of certainty obtained in the generalisation. The most perfect induction is one that leads by strict logical inference to a certain conclusion, or a true universal synthetical proposition, or a law of nature. Thus, complete enumeration and colligation of facts are types of improper but perfect induction. Likewise, there can be no doubt that scientific induction affords complete certainty, not only in its practical results, as is clear, but also in theoretical reasonings. Some of Mill's inductive methods and the Physical Method are examples of proper and perfect induction.

7. Examples of Induction. The reader may examine from a logical point of view the following accounts of scientific inquiry. And let him make out distinctly (*a*) what the problem is, (*b*) the hypothesis or hypotheses put forward on the evidence of facts observed, (*c*) the method of verification, and (*d*) whether the conclusions are probable or certain. It is interesting to see in the course of analysis the great difficulty, often referred to, of actual research. Moreover, one can realise how the scientist proceeds in his own ways, which are not so simple as the logical forms of the methods seem to imply.

I. **Odoriferous Substances.**[1] Certain bodies have the property of constantly giving off to the atmosphere extremely minute particles of their substance. These tiny particles, although much too small to be perceived by either the organs of touch or taste, are readily appreciated by the organ of smell, and produce what we call an *odour*.

The fact that smells are transmitted through space like light and sound has suggested the possibility that they may depend upon a vibratory movement of some medium. This hypothesis, although occasionally defended in modern times, is apparently entirely incompatible with experience. The usual view is that odoriferous bodies emit particles which, as a rule at least, are in gaseous form. Smells are absorbed by certain substances such as water, milk, resins, oils, blotting-paper, wool, etc., even without actual contact with the odoriferous body. The smell of tobacco clings to woollen curtains, while hay soaked in water absorbs the smell of paint. You know how butter, set beside fish, in the ice box will get fishy flavour.

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[1] Adapted from original work by Rev. Father J. F. Caius, S.J., Pharmacological Laboratory, Bombay.

The fineness of the particles is remarkable, because if the air conveying an odour be filtered through a tube packed with cotton wool and inserted into the nose a smell is still discernible. A grain of musk or of vanilla will scent an apartment for years, and in the end of the time no appreciable loss of weight can be detected. Nevertheless the particles emitted are material. Repeated washing will fail to remove the smell from a fabric charged with vanillian vapours ; but the moment we treat the tissue with bisulphite the odour will instantaneously vanish. The experiment not only proves that odoriferous bodies emit material particles, but also that the sense of smell is a chemical sense. The unaided nostril can rival the spectroscope in the detection and analysis of unweighable amounts of matter.

The sensations of smell are aroused, not by the transmission of wave-motions through air or ether, but by the actual contact of material particles with the sense-organ. The odorous particles may be given off by volatile substances in our immediate surroundings, or may be brought from a distance by currents of air. They are received into the nose in the act of inspiration ; if we wish to get the full fragrance of a flower we sniff at it ; so long as we hold the breath, we smell nothing. In short, the particles are conveyed to the olfactory epithelium by currents in the air or by simple gaseous diffusion, and after solution in the moisture of the membrane act chemically upon the sensitive hairs of the sense cells. Thus the scent of flowers and the stench of putrefaction are due to vapours of chemical substances.

All vapours or gases, however, are not capable of acting as stimuli to the sense-cells ; so that evidently the odoriferous character depends upon some peculiarity of structure. There can be no doubt that the action of stimuli upon the organ of smell is chemical in its nature, so that a substance is odorous or inodorous by virtue of its chemical constitution. In fact it is always possible to point to the existence of active atoms in the molecule of an odorous substance. Smell leads us to the heart of the molecule and enables us to tell how the atoms are put together. A chemist, given an unknown substance, would have to make an elementary analysis and some tedious tests to determine whether it contained methyl or ethyl groups, whether it was an aldehyde or an ester, whether the carbon atoms were singly or doubly linked, and whether it was an open chain or closed. But let him get a whiff of it, and he can instantly give a pretty shrewd guess as to these points. His nose knows. In general it may be said that most odoriferous elements are found in the fifth, sixth, and seventh groups in the system of Mendeleeff : they are elements with varying valencies or residual affinities.

Such elements as oxygen, nitrogen, sulphur, phosphorus, the halogens, which have varying valencies, are all capable of forming odoriferous groups. Carbon and hydrogen, which possess no residual affinities, produce little or no odour. We are thus led to assume that there are certain groups which are characteristic of all odoriferous substances, and by virtue of which substances react with the special form of protoplasm found in the hair cells.

Ruzickie postulated the existence, in the lipoids of the mucous membrane, of a grouping, the "osmoceptor," which is capable of definitely uniting with some group in the odoriferous substance and forming in this way a complex. The resulting disturbance of the energy equilibrium acts as a stimulus and produces a sensation which is perceived by the nervous centre.

The molecules of the odoriferous body at first combine with the most active osmoceptors, the "primary osmoceptors," and a well-defined characteristic smell is produced. When the affinities of all the primary osmoceptors have been satisfied, other groupings, the "secondary osmoceptors," may enter into play, and there arises another smell different from the first. This would account for the fact that many vapours, indol, ionone, etc.,—which produce a pleasant perfume when freely diluted with air, become an unbearable stench when too highly concentrated. No one of you would want the undiluted natural oil of violet if you could get it, for it is unendurable. A single whiff of it paralyses your sense of smell for a time, just as a loud noise deafens you.

The osmoceptor theory explains equally well the phenomena of fatigue and adaptation. The nose may be fatigued for one odour and remain sensitive to others: after one has smelled camphor for some time, alcohol will not be noticed, but iodine will still have its effects. The student of chemistry and anatomy ceases in a short time to react to the numerous odours surrounding him. Individuals in large audiences housed in poorly ventilated rooms become adapted to the various odours and perfumes. Individuals just coming in react strongly to this situation.

As in the case of all sensations, the only direct knowledge we can obtain of olfactory sensations is from introspective examination of our olfactory experience, and introspective examination is, in the case, particularly difficult. There are some people who seem entirely devoid of a sense of smell; there are others who are incapable of perceiving certain smells. Many people, for example, are unable to smell the odour of prussic acid; others are unable to smell benzoin or vanilla; for others mignonette has no smell, nor have violets. Not only are these great individual differences with respect to the degrees of sensitivity at different times; it has been estimated in the case of "Heliotropine" that the same individual could be fifty times less sensitive from day to day.

The uncertainties of science are reflected in the popular speech. There are no names for odours other than those of the objects that give rise to them; we speak of the smell of onions, of a lily, of fish, and so on. We have no special names to designate special odours and to distinguish one odour from another, with the exception of the general terms "pleasant" and "unpleasant." The difficulty is increased by the fact that tactual and taste qualities mix with the olfactory. The sweet odour of chloroform is really a taste. The odour of ammonia is largely pain, and the resulting holding of the breath adds a feeling of suffocation.

When all has been said, we know little about the nature and number of adequate olfactory stimuli. The various odoriferous substances have been classified with reference to the general similarity of response produced by them; but the best of these classifications has little to recommend it.

Nevertheless, as in the case of other sense departments, we can determine thresholds, both absolute and differential, for odours. By taking known amounts of odoriferous substances and diluting them to known extents it is possible to express in weights the minimal amount of each substance that can cause a solution. By this method such figures as the following are obtained : camphor is perceived in a dilution of 1 part to 400,000 ; musk 1 part to 8,000,000 ; vanillin 1 part to 10,000,000.

II. **Earth-eating and Salt-licking.**[1] "Geophagy or earth-eating is a matter on which much has been said and written. The subject has been much laboured but little advanced, unless we agree to use the term advance for the propounding of not less than half-a-dozen theories to explain this 'strange' habit.

"Earth-eating is indulged in by both the sexes, at any time of life, in all states and grades of society. Moreover, the purpose differs no less widely than the condition of the eaters as regards sex, age, or health before and after acquiring the habit. Earth is eaten as a delicacy, as a surrogate for confectionery, as a condiment, as a recognized article of food, as a medicine, as a tonic, as a charm, or as the result of imitation. Again, we find the most extraordinary variety of material used. Not only are clays, shales, and alluvial muds commonly taken ; but, with the exception of sand, earth of almost every description is eaten.

"Earth-eating has been referred to as a strange habit, an unnatural habit. Strange, no doubt, when we consider man's usual bill of fare. Is it then unnatural in the sense of nutritional abnormality ? We do not think so. The purport of this inquiry is to show that this widespread craving for earth of some kind is the psychological expression of a physiological need, that the habit primarily rests on a physiological basis.

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[1] From a paper by J. F. Caius, S.J., and K. H. Bharucha, B.Sc., Pharmacological Laboratory, Bombay.

" The notion of food is natural. It belongs to that common sense physiology which is transmitted from parent to child by heredity, and which the scientific fraternity find a difficult task to alter. We eat by instinct and habit, and it is instinct and habit which prescribe the menu. But whereas wild animals select their food naturally, the food of civilized man and of domestic animals is provided and prepared artificially.

" It is generally admitted that an adequate diet must contain an adequate supply of the various inorganic substances which enter into the structure of the body. In contrast to the ' organic ' nutrients, which act primarily as carriers of energy, the mineral ingredients of food introduce no energy into the organism but do serve important purposes, both structural and functional, which we shall consider and discuss later when we bring this inquiry to a close.

" The human body contains approximately : 2 per cent. calcium, 1 per cent. phosphorus, 0·35 per cent. potassium, 0·25 per cent. sulphur, 0·15 per cent. sodium, 0·15 per cent. chlorine, 0·05 per cent. magnesium, 0·004 per cent. iron, and very minute quantities of iodine, fluorine and silicon.

" The dietary adequacy of the last three elements is left to chance. Iodine, however, is an essential constituent of the thyroid gland, and it seems probable, from recent investigations, that an adequate supply of iodine in the food is essential for the normal functioning of this gland. The presence of iodine is moreover inseparable from certain periods in a woman's life, and these are precisely the periods when, in their case, the earth-eating habit is contracted.

" The sulphur requirement is met by the proteins—

the nitrogenous compounds of food—of which it constitutes about 1 per cent. In practice, when the protein is adequate, the sulphur has not been found deficient.

"If a reasonable amount of vegetable diet is used, the potassium requirement is fully met ; and the universal practice of using salt as a condiment insures a sufficient supply of both sodium and chlorine. Since salt is constantly being eliminated by the skin and kidneys it must be supplied by the food, and the reason for the addition of salt to potatoes and other vegetables is physiological as well as condimental.

"The magnesium requirement has not been thoroughly studied ; but it seems to be adequately supplied by the mineral content of vegetables and meats, in many of which it is present in greater quantity than calcium.

"The inorganic elements that are most likely to be deficient in a diet are calcium, phosphorus, and iron. We are inclined to attribute earth-eating among men and salt-licking among wild animals primarily to a deficiency in one or more of these elements. A superficial examination of the soils sought after by wild animals points to the fact that they are closely related to, if not the same as, the materials used by earth-eaters. And one thing which stands out prominently is that, whatever it is the wild animals seek in the salt-licks, it is not sodium chloride. What it is may be revealed by a careful analysis of numerous samples of soils from the tracts visited by them. This is to be the first step in our inquiry."

CHAPTER XV

PROBABILITY

1. **The Purpose of Probability.** The study of Probability has a definite place in the science of Logic as an extension and a lower kind of material inference. In fact Probability helps to Induction where all other methods and even hypotheses fail.

There are many facts that have no definite connection with causes or antecedent conditions, and yet need to be generalised. Some of these generalisations assist us when we have to make up our mind about our daily affairs; other generalisations afford some sort of a guide to solve social and physical problems. They help us to *practical* decisions when the evidence is not sufficient to give us *absolute* ones. An attempt to arrive at propositions more or less general from disconnected facts is the purpose of Probability.

Phenomena disconnected from their causes are very numerous and of various kinds, but all may be arranged under three heads, namely:

(*a*) Those which cannot be reduced to ascertained causes, and have to be analysed extrinsically in order to obtain a guiding rule as to their occurrence. Examples of such events are motor accidents, shipwrecks, bad crops, death from certain diseases, death of individuals at different ages, successful voyages, business results, etc.

(*b*) Cross-events or causes that concur together in producing an effect; in which case we have the problem of combined probabilities, leading to the final probability of a given fact.

(*c*) Consideration of errors which necessarily occur in all our measurements and observations in order to ascertain the most probable magnitude. In regard to errors it is known that two or more observers will not always arrive at equal results. In that case what is the most probable value? The problem before us is how to reach in our measurements the most probable law or the most probable magnitude.

The first two cases are attempts to eliminate chance, the third aims at reducing errors.

2. Chance and Contingency. Before coming to define Probability it will be well to explain the notions of chance and contingency which are implied in that definition.

Chance means an event happening without any calculation or expectation on our part; as when a man walking home happens to see a jewel lying on the ground. Another name for chance is *coincidence*. It means (1) two things happening *together* without any causal connection between the two, or any common *higher* cause controlling the two lower causes so as to bring the two events on at the same time. For example, two friends meet at the station, one coming from Poona, the other going to Poona, each for his own reasons. Or else (2) it means two things happening one after the other, without the one being the cause of the other.

Examples : I get smashed up by a motor-car, and an ambulance wagon happens to pass a minute after, not summoned for the accident. Or a man quarrels with his bishop, and happens to die of cholera shortly after.

Things of that kind are called chance, and are most naturally explained away by the expression " merely a coincidence " to signify that there is no ground to infer recúrrence according to law. We should remark, however, that in reality there is no chance. Every event is really determined by definite circumstances, because every event must have a cause ; but as we are ignorant of the cause, we cannot make calculations about it. If we knew all the conditions of natural phenomena as they are in themselves, chance would disappear. The vulgar conception of " fate " as inevitable necessity due to some mysterious force is out of the question in science.

Contingency is another name for chance in popular language ; but in philosophy it means a thing that *might or might not happen*, as distinguished from what *must* happen. The notion of contingency is *objective*, and is distinct from *doubt* which is *subjective*, i.e. a psychological effect of the thing being contingent.

3. Nature of Probability. The word probability is ambiguous. (1) In its Latin origin (*probabile, probabilitas*), it means capable of proof, and that there is some evidence for a thing or event, but not so clear or conclusive as to demonstrate it with certainty. (2) But in common language the word *probable* rather means that it is likely to be ; that is, a sort of thing one can rather expect without being sure that it will happen. (3) In our logical discussion probability fills up the space between two extremes : *certainly*, and *certainly not* ; between what is bound to be a fact, and what is not and cannot be a fact. In other words it covers the range of concrete possibility, that a thing *may* be or *may not* be, because in the abstract it *can* be. If the possibility seems *likely*, we say it is *probable*, *very probable*, *somewhat probable* according to the degree of evidence suggesting it. If it seems *unlikely* we say it is *improbable*, *very improbable*, *somewhat improbable*. The criterion is always some kind of evidence suggesting that the thing *may be*, but not proving conclusively that it is or will be. Perhaps objectively it is or is not, but we do not know either way ; and so we adopt the middle attitude of suspension of mind, with a tendency to expect that *it is* or *is not*, according to the suggestions of dubious evidence. Whenever an event is in the position that it may just as well happen as not, the question arises as to what we ought to expect. In order to clear this question, let us begin by a definition.

Probability is the measure of our rational expectation in regard to the happening of an event that has no known connection with a cause. For convenience of expression it is represented by a fraction running from 0 or *certainly not*, to 1 or *certainty*. Observe that what we measure is not subjective belief, but objective credibility. The former is supposed to follow from the latter, and this for the following reasons :

(*a*) It is the property of a healthy mind to have a *reason why* we think it probable, and this reason must be some objective consideration of fact *pointing that way*, and not a mere *wish* or fancy.

(*b*) Subjective belief is a state of the mind, psychological in nature, that cannot be satisfactorily measured because it has no definite value. On the contrary, it is changeable in the same person and may differ in different persons. It is often seen in games of chance, where one man draws from a bag, but soon gives up in despair, while another in the same circumstances keeps up his hopes.

(*c*) Another reason why we do not try to measure subjective belief is that Probability, as part of inductive Logic, must rest on grounds of observation or on accepted principles. We aim at verifying and proving, and this is done by reference to those grounds.

We said that belief follows from credibility, meaning by credibility such perception of facts as inspires belief. This credibility produces in the mind various degrees of belief, and accordingly we say that an event is *possible, very improbable*, *probable*, *very probable*. These are the judgments which facts, as presented, enable us to form, or judgments which the average man *ought* to be able to form in the presence of facts. It may be objected that different minds may take different views of the same fact ; but this is immaterial to our study. A rational expectation is the kind of expectation which arises in the average mind. A comparison will make this clear. By a thermometer we measure the temperature that will make people comfortable in a hall. There may be among them feverish persons who feel uncomfortable, yet this anomaly does not change the true temperature nor the general comfort following from it. In like manner we measure the credibility of facts from which follows the average expectation.

4. The Grounds of Probability. We cannot speak of formal grounds or uniformities like those employed in scientific induction, for there are none in Probability. Again there are many kinds of events, physical and social ; consequently the knowledge available is also very different. However, the evidence employed as a basis of our expectation may be reduced to three forms:

(1) An estimate of the frequency of the event by calculating the prevailing conditions or causes in favour and against—in other words, a proportion of favourable cases given *a priori*. Thus we know that a coin tossed will turn head or tail. Similarly a die at one throw can turn any one of the six faces, and the probability of it being ace is one out of six. If a bag contains ten black balls and twenty white, the probability of drawing black is 10 out of 30.

(2) An estimate of the frequency of the event by observing the number of cases for and against the occurrence of the event, without any knowledge of the operative conditions—in other words a proportion of favourable cases given *a posteriori*. This method is the only way open in pure statistics to arrive at some sort of general knowledge. Thus, for instance, the probability of a successful election, of accidental death, of an efficient medicine, of passing an examination, of the life-period of an individual, etc., is based on observed frequency of favourable cases.

(3) The two methods together, that is, frequency as given by previous knowledge *a priori*, confirmed by a frequency of occurrence observed *a posteriori*. The best types of probability usually present these two aspects. Thus the probability of a die turning ace is by calculation $\frac{1}{6}$, and the number of times found by observation is about 16.6 % of the number of throws, but never exactly.

It is plain that the third method is more reliable than either of the other two, because based on conditions that imply more or less uniformity. The rules of life or shipwreck insurance are often baffled by unexpected outbreaks of epidemics or unusual storms.

We have mentioned two ways of measurement, the *a priori* and the *a posteriori* aspects of probability in a series of facts. In practice it is often legitimate to infer the hidden existence of one by the manifested presence of the other. The cogency of the inference will depend upon the amount of knowledge. In the case of a perfect coin we feel sure from the conditions known *a priori* that in the long run of throws the number of heads and tails will tend to be equal. And vice versa, if a jar contains black and white balls in unknown proportion, sufficient experience in drawing, putting back the ball each time, will suggest the proportion in which they are mixed. Again, if I measure a pint of water fallen on a square foot during a shower, I may suppose the same amount has fallen on each square foot of the neighbouring ground. If ten students out of a hundred are absent from College on account of sickness, I may suppose that the same proportion of sick people exists among the surrounding population.

5. How the Measure of Probability is Ascertained. The following views concerning simple and combined probability have been generally accepted after the manner first proposed by Laplace.

Rule I. The probability of an event is the ratio of the number of cases which favour it to the number of all possible cases, when nothing leads us to believe that one of these cases ought to occur rather than the others, which renders them, for us, equally possible. This first principle builds up our rational expectation of an event of which the operating cause is ignored. Data may be gathered *a priori*, *a posteriori* or in both ways. Examples :

1. If there be ten balls in a bag, of which one is white and the rest black, at the first draw the probability for white is $\frac{1}{10}$ and that for black $\frac{9}{10}$. The odds against white are 9 to 1, and for white 1 to 9. The sum of the chances for and against must be the total number or $\frac{10}{10}=1$, certainty. In the long run the same ratio will be confirmed by experience.

2. Similarly the probability of catching an epidemic disease is a ratio, but the data in this case are only from experience. If the people number ten thousand, and those attacked daily are ten, the probability for each person will be one out of a thousand. We go on the supposition that there is no reason for one rather than another, or very nearly so. If this condition fails the rule also fails.

Rule II. If two events do not concur, the probability that either one or the other will occur is expressed by the sum of the separate probabilities. Examples :

1. The probability of a die turning either ace or six at two successive throws is $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$.

2. The probability of dying of one of two diseases raging in town is equal to the ratio of the probability of the one plus the ratio of the probability of the other, according to data from experience.

3. The same rule holds good when several distinct conditions have the same consequence. Thus, death may occur on a voyage by sea either from shipwreck or from fire on board. Statistics being given for each case, the probability of meeting death from either one of the two causes will be equal to the sum of the two separate probabilities.

Once the first rule is accepted there ought to be no difficulty in the second, which is an extension of the former. For, if the favourable cases that make the probability or danger of a disease are divided into two groups, the total danger or probability will be the sum of the dangers or probabilities appertaining to each group.

Rule III. The probability of the concurrence of two events independent of each other is the product of the probability of the one event mutiplied by the probability of the other. Examples :

1. The probability of two dice both turning ace at one throw is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. In this case and in similar cases of chance games, the assumption is verified by experience. But the principle can also be applied to instances that transcend verification.
2. If a student sits for examination in several subjects, the probability of passing in all is the product of the separate probabilities.
3. If members of various communities stand as candidates in municipal elections, the probability of two Parsis being elected is the product of both probabilities.

Observe that the same rule holds good if the two events are inter-dependent. Examples :

1. The probability that St. Xavier's team will win the cup, competing with two rival teams, may be estimated as equal to the probability of winning at the first competition multiplied by the probability of winning at the second, where the second victory is conditioned by the first.
2. Laplace exemplifies the composition of such probabilities as follows : Suppose three urns, A, B, C, about which it is known that two contain only white balls and one only black balls. The probability of drawing a white ball from an assigned urn, say C, is $\frac{2}{3}$. The probability that, a white ball having been drawn from C, a ball drawn from B will be white is $\frac{1}{2}$; therefore the probability of drawing a white ball from C and also from B, or the probability of the double event is $\frac{2}{3} \times \frac{1}{2}$ or $\frac{1}{3}$.

Once the principle is accepted, it follows that in the case of a double event the probability of one of them will be obtained by dividing the product by the probability of the other.

The Rule of Succession. This rule refers to the probability of the recurrence of an event that has happened a number of times. It may be stated as follows : To find the chance of the recurrence of an event already observed, divide the number of times the event has been observed increased by one, by the same number increased by two. Laplace calculated by this rule the probability of the sun rising to-morrow. De Morgan employs the following example :

> If a man standing on the bank of a river has seen ten ships pass by with flags, he will expect that the next ship will also carry a flag according to the ratio $\frac{11}{12}$. The principle is an attempt to estimate probability in induction by simple enumeration. Its value is controverted.[1]

6. Inverse Probability. No new principle is involved in what is called retrospective or inverse probability. This process consists in inferring the probability of a cause from an observed event ; that is to say, in transferring the probability observed to the cause or causes. At times from causes we descend to collateral effects ; or again, from probability observed we argue to future events. The whole is a matter of probable reasoning in a great variety of subject-matters. Examples will make it plain.

1. If a die turns six oftener than it should, there is reason to suspect some load operating in favour of that face.

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[1] See Jevons, *Principles of Science*, p. 257.
Venn, *Logic of Chance*, p. 196.
Keynes, *A Treatise on Probability*, p. 382.

And in general, if the occurrence of an event is more or less frequent than estimated probability suggests, it is reasonable to suppose the existence of connection with a separate condition or cause.

2. The probability that iron exists in the sun is inferred from comparing the spectra of sunlight with that of the light proceeding from the incandescent vapour of iron. In like manner from the spectrum of earthly materials Physicists argue to the probable constitution of the stars.

3. From statistics regarding kinds of books demanded by the general public from a circulating library, say 80% novels, I may conclude to the same proportion of novel-reading people in the community.

4. If I observe a fall of attendance at College on Mondays and Saturdays, I may expect that the same will happen in other Colleges and in the future.

5. From the observed danger of the road traffic in cities and in the country, we argue to the probable causes, namely (*a*) Error or negligence on the part of the drivers ; (*b*) Mechanical defects in the vehicles, or unsatisfactory road conditions ; (*c*) Error or negligence of persons other than drivers.

6. Statistics show that in the post-war period the trade of the world has not kept pace with the growth of its production. This experience points to the probable sources of this trouble.

Observe that in all cases of inverse probability the cogency of the inference depends upon and varies according to the degree of experience. The presumption must be extended with caution to phenomena less familiar or particularly complex. Thus, for instance, if earthquakes have occurred in a certain place, we reasonably expect that they may occur in the future. But because meteors have fallen on a certain place, we have no reason to suppose that they will or will not hit that place in the future. Still more risky is inference on social matters where the factor of free will is involved.

7. Probability in Measuring. Another kind of probability is that extensively used for finding the true magnitude of a phenomenon by appealing to the average.

An average may be defined as a quantity derived from a given set of quantities such that, if those quantities are reduced to equality the average will represent their common measure. But since those quantities are unequal, the average is greater than the least, and less than the greatest.[1]

Observations or measurements made by different people often disagree due to errors among them, an error peculiar to each. The question then arises : How to eliminate error ? **An error** may be defined in general as a deviation from truth ; in our case, a deviation from the true magnitude of a phenomenon. Errors are bound to occur in our observations, as neither our senses nor our instruments are exact. Our senses are not precise in appreciating minute quantities, and instruments vary according to temperature and other atmospheric conditions.

Attaining the Mean. Constant errors from friction, temperature, personal equation, etc., already known, are easily eliminated by addition or substraction, according as they are in excess or defect of the average. We suppose then a series of observations out of which we want to discover the true one or the average. Here probability comes in. We begin by the supposition that errors may occur in opposite directions, and with equal chances, as we have no reason for one rather than the other direction. For the same reason, we also suppose that errors by excess and by defect will be so distributed as to balance each other. This is to be understood within a limited range or amount of error; greater errors are few and may be discarded. From these suppositions it follows that the best observation available is the mean of them all, or what is called a reduced observation. The arithmetical mean is the most common, namely, the sum of all the observations divided by the number of them.

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[1] For a full exposition of this matter see Jevons, *Principles of Science*, Chapter XVI.

Average and Mean are not synonymous. The word "mean," says Jevons, denotes approximation to a definite existing quantity. When the mean is a fictitious quantity used for convenience of thought and expression it should be called an "average."

There are other means, besides the arithmetical; for instance, the square root of the product used for quantities that vary according to the square of the distance. These methods are very practical in Physics and other material sciences, and may be used in testing inductions, in improving the Method of Concomitant Variations, and in reducing the long labour of statistics.

8. Probability and Mathematics. A treatise on probability involves mathematical calculations regarding the enumeration of cases; that is to say, finding out the proportion of favourable cases to the total number of cases. The operations often imply permutations and combinations expressed by corresponding formulas; in fact any amount of mathematical speculation may be used in estimating probability. Our purpose in Logic is limited to the study of the most primary axioms or suppositions on which those calculations are based, and to the probability derived from them. It is noticeable that the mathematical calculations are in themselves clear and certain, while the suppositions on which these calculations are based in probable matters are obscure and uncertain.

9. Probable Arguments. A probable argument is one that leads to a probable conclusion. It is called **self-infirmative,** if the conclusion becomes weakened by one or more probable premises; the argument is called **self-corroborative,** if two or more separate arguments are used to prove the same conclusion, each one being only probable. The question is how to estimate the amount of probability.

The rules suggested are much the same as those given for the probability of an event. As the conclusion is inferred from both premises, we may suppose that the extension or probability of the one is combined with that of the other. Hence:

(*a*) If one premise only is probable, the conclusion has the same probability.

(*b*) If both premises are probable and their probability is quantified, the conclusion will be weakened according to the multiplication of the two separate probabilities. Examples:

1. Graduates in English Universities are likely to be employed by Government—probably as $\frac{2}{3}$. My brother may be a graduate in English Universities—probably as $\frac{3}{4}$. Therefore: My brother is likely to be employed by Government—probably as $\frac{1}{2}$.

2. In a similar way is measured the way in which truth deteriorates while passing from one to another by hearsay. If in first-hand information a witness is valued say at $\frac{3}{4}$, the truth of the second who receives it from the first will be $\frac{3}{4}$ of $\frac{3}{4}$.

In **self-corroborative arguments** we have to estimate the probability of the conclusion as increased by the separate probabilities; that is to say, by the strength of each argument. The rule given by some authors is as follows: Multiply together the fractions representing the improbability of each and subtract the product from unity. Examples :

1. Two arguments, A and B, prove separately the conclusion C.

Probability of A ($\frac{3}{4}$), improbability ($\frac{1}{4}$),
Probability of B ($\frac{2}{3}$), improbability ($\frac{1}{3}$),
Probability of C ($1—\frac{1}{4}\times\frac{1}{3}$) or $\frac{11}{12}$.

2. A practical instance would be to estimate the probability of a statement or fact as affirmed by persons who seem to know the truth about it.

10. Analogy. Another kind of probability is that based on analogy between two things. Analogy means resemblance or similarity among things which are otherwise dissimilar ; or in the concrete, things are said to be analogous when they are partly similar, although also partly different.

Here we must distinguish between metaphorical and real similarity. The former is rather far-fetched and produced by our imagination, as when I say : Human passions are as quickly inflamed as gunpowder ; Market prices fluctuate like the tide of the ocean. Such similarity, being vague and weak, will not stand the test of a logical argument ; its place is rather in Poetry and Rhetoric than in Logic.

Real analogy consists in definite characteristics discovered in the very nature of objects and specifically the same in them. Such similarity may serve the purpose of an argument, which consists of an inference from similarity in some qualities to similarity in other qualities. Suppose **A** and **B** are similar in some respect to **X.** But we know that **A** has not only the property **X** but also the property **Y**--perceived by us. Since **B** also has the property **X**, it is quite likely that it may also have the property **Y**, though the property **Y** is not yet discovered by us in **B**. Examples :

1. The two planets Mars and Earth are found to be similar in their revolution around the sun, in temperature and atmosphere ; therefore both may be similarly inhabited.
2. Two brothers have had a similar home education ; that is to say, from the same parents, with the same care and means ; therefore the second will be successful in life like the first.
3. The countries of Southern India and Ceylon are similar in latitude, heat, rain and other atmospheric conditions to the Peninsula of Malacca and the island of Sumatra ; therefore the productions that grow well in the former, say, coffee plantations, will also grow well in the latter.

In these examples the conditions are the same ; there is real similarity, not a metaphorical one like that between inflammation of the passions and of gunpowder.

11. The Ground of Analogical Arguments. The ground is an assumption that things similar in one respect will be similar in another, which is merely probable, as may be seen by comparison with other grounds of Induction.

To begin with, this ground cannot compare with the syllogistic ground, which is identity or a necessary relation of the extremes with the middle term. The methods of Induction rest on the signs of a cause, leading us to the proofs of causation or uniformity ; but these signs are absent in analogy. In the case of simple enumeration, where we pass from uncontradicted experience to a generalisation, there is a certain ground for supposing uniformity—only probable indeed, but still a ground. Even this much is lacking in analogy. At the most we may say that the greater the number of similar qualities in both objects, the stronger the inference based on them. Analogy looked at in this way resembles enumeration of instances. We conclude, therefore, that the ground of an analogical argument is essentially probable.

12. How to Estimate the Value of Analogy. Not all arguments of this kind are equal in probability. The general rule is that we should weigh the amount of similarity, looking into its nature as bearing on the inference. Take, for instance, the example given above to prove that Mars is inhabited. The points of similarity between the two planets Mars and Earth are remarkable ; but when examined in reference to the possibility of life, we have to look for a number of other conditions. The argument would increase greatly in probability, if signs of vegetation could be discovered in Mars.

The particular rules to estimate the cogency of analogical arguments are the following :

(*a*) Probability increases with the number of points of similarity, specially if they are so important that they may be the source of other qualities. Thus a very proficient student in Law is likely to be successful in his profession.

(*b*) The greater the number and importance of the points of difference, the weaker is the inference. For instance, to argue that I will be successful in life because my elder brother or my father were successful, carries little probability. The points of difference, viz. personal qualifications, are most important.

(*c*) The improbability grows with the number of qualities unknown in both objects. To say, for example, that a law of prohibition passed in the United States should also be enforced in England because both are English-speaking countries, has no force if we ignore other conditions.

13. Usefulness of Analogy. We have seen that Analogy is essentially weak for the purpose of argument. It is nevertheless very useful as a help to investigation. It affords a means of illustration in obscure and difficult matters, showing a way of action or a result otherwise unintelligible. Thus, nervous actions are illustrated by comparison with electricity or magnetism.

Furthermore, Analogy is valuable for suggesting hypotheses. Real points of similarity indicate possible connections with other qualities and effects ; in fact, an argument from Analogy is often in itself an hypothesis.

14. Kinds of Argument from Analogy. The general argument from Analogy is that similar qualities or conditions may be accompanied by similar effects or similar qualities. This inference may assume various forms :

Argument "a pari." The Aristotelian analogy was a perfect type of this argument. He meant by Analogy a proportion or equality of two ratios. Observe here that, if the ratios have a proved relation, the argument is certain ; for instance, when I say $\frac{2}{4}=\frac{3}{6}$. But analogical proportion is based on similarity, as in the following : Health is to the body as Virtue is to the soul ; As sound is to the air, so light is to the ether ; A medicine that produces good effects in a quadruped will produce the same in a man, who is likewise an animal.

Argument " per exemplum " is also *a pari*. Orators make good use of a simple example to bring about persuasion ; its force, however, depends on the power of description and emotion. For example : A man was ruined for life on account of alcoholic drink ; therefore other men will be similarly ruined.

Argument "per contra" means arguing from opposite qualities or effects to opposite results or conditions. Examples : Uneducated people take to menial work, therefore educated people will embrace higher pursuits. Heat expands bodies, therefore cold contracts them.

Argument " a fortiori " in which we infer an increase of a certain quality or effect from an increase in similar effects or qualities. Examples : If strong people are affected by the climate of Bombay, the more so are the weak. If parents must look after the bodily welfare of their children, with greater reason should they see to their intellectual education.

Analogy and Syllogistic Form. We may apply here what has been said of probable arguments in general, namely, that such arguments can never be thrown into syllogistic form. The Enthymeme, in which one premise is omitted, is most natural for analogical arguments. If they are stated in full syllogistic form, the inconclusiveness will be evident by the presence of four terms. The reason is because in those arguments there is no perfect inclusion of one term in another, nor a general premise producing a true general connection.

CHAPTER XVI

CLASSIFICATION

1. The Purpose of Classification. Is Classification a process of Induction ? And it so, what does the process lead to ? An answer to these questions will tell us the purpose of Classification.

As to the first question, it will suffice to say that by examining individual objects and reducing them to classes we generalise ; that is to say, we arrive at general names. This process is not as perfect as that by which a general proposition is obtained. I say as perfect ; for names afford the notions of things, or the materials of science, while a general proposition binds them together in a common relation within a system of knowledge. But in a less perfect way and by analogy of process Classification is a kind of induction, and prepares the way for other inductions. Before an attempt is made to reach a universal connection or a law of nature, it is indispensable to know the meaning of the names to be related.

Coming to the second question, Classification leads to the knowledge of the connotation of names ; that is, to distinguish what is essential and what is accidental in them. Observe that in deduction we took for granted that connotation is given to us ready made ; but in Classification we build up that connotation. Knowing what is essential and what accidental in the objects, we come to know their relationship. The purpose, therefore, of Classification is :

(*a*) To build up the connotation of names.

(*b*) To place in a clear light the relationship that exists between them.

To give an instance, the ninety-two simple elements, recognised in Chemistry as the constituents of all material substances, have been found, both as to what they are, and how they are related, by a long process of Classification that has occupied the greatest chemists down to the present day.

2. Nature of Classification. To classify is to separate individuals into groups according to their resemblance and difference ; carrying the process by degrees from the specific groups to wider classes, till we arrive at one general group embracing them all. Grouping according to resemblance and difference means that objects most alike go together, while those that differ greatly are put aside in another group. This separation may be made by dychotomy or by positive terms ; but in every case we must proceed on a common basis ; that is to say, a definite characteristic at each step.

As an illustration of this activity of the mind, take three objects **A B, A C, A D,** where **A** stands for important characteristics common to all three, while **B, C, D** are essential differences. The common characteristics **A** are readily locked up in one notion and constitute a *genus* of the three *species* **A B, A C, A D.** This genus **A** is less in connotation than any of the species, but in extension embraces all the individuals under those species. By a further analysis of the characteristics **A** we may discover a common element among other objects and arrive at a wider class. To make it concrete in a familiar example: All triangles agree in having three sides, but differ in the nature of angles; hence the three kinds, acute-angled, obtuse-angled, right-angled; three *species* of the *genus* triangle. Now a triangle is a plane figure limited by three sides; and there are plane figures limited by four, by five, etc. Plane figure, therefore, is a wider class embracing triangle, quadrilateral, etc.

Generally speaking, "by the classification of any series of objects is meant the actual or ideal arrangement together of those which are like and the separation of those which are unlike, the purpose of this arrangement being, primarily, to disclose the correlations or laws of union of properties and circumstances, and, secondarily, to facilitate the operations of the mind in clearly conceiving and retaining in the memory the characters of the objects in question." [1]

It is plain that the same objects may be classified in various ways. Books, for instance, may be classified by authors, by languages, by the subject-matter, etc. The process is a slow one, involving a thorough analysis of the objects, which may consist in mere observation, or, if the classification is scientific, may require experimental research. Having thus examined every object in turn, we compare the result to find out in what and how much they agree and disagree.

3. Artificial and Natural Classifications. We said that the same objects may be classified in various ways; our purpose in view will decide the basis of classification. When our view is practical, corresponding to a particular end or use, the classification is said to be **artificial,** for it depends upon our selection. Pictures in a gallery, monuments of antiquity, automobiles, in fact, any set of objects may be considered according to author, price, shape, etc. Any exterior mark may be the basis of artificial classification, but a natural quality is better than a merely arbitrary one.

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[1] Jevons, *The Principles of Science*, p. 677.

Natural Classification, also called **scientific,** aims at the knowledge of things, and is based on characteristics that constitute their nature, or at least are deeply rooted in them. Examples :

1. The chemical classification of material substances according to the essential elements that make up the compound is natural.
2. The classification of books by the nature of contents according to the decimal system is natural.
3. In Zoology the classification of animals into vertebrates and invertebrates, etc., is natural.
4. The classification of students into undergraduates, graduates and post-graduates is artificial.
5. In Meteorology the general classification of clouds is naturally reduced to four fundamental groups : Cirrus, Stratus, Cumulus, Nimbus.
6. In Geology all kinds of stone come naturally under such groups as Sandstone, Limestone, Granite ; or in other words : Sedimentary Rocks, Organically-formed Rocks, Igneous Rocks.
7. Stars are classed by their brightness from the first magnitude to the sixteenth, a very simple artificial classification.

Natural classifications, being grounded on the nature of things, are universal and suitable for the purpose of science. There each group has its proper characteristics, definite meaning and name ; while their gradual subordination shows the order, harmony and unity existing amidst the great variety of objects in nature. Thus the purpose of classification, proposed above, is attained.

Every science possesses its own classification, but the natural sciences, Mineralogy, Botany and Zoology, that study the individual forms of nature, depend entirely on classification and are called "classificatory sciences." The ideal of a universal classification, broad enough to embrace the subject-matter of all material sciences, is still beyond the reach of our present knowledge.

4. Systems of Classification. If natural classification consists, as we have said, in grouping things according to their essential characteristics, there seems to be no room for theoretical systems. There are, however, opinions as to the way to be followed.

Classification by Types. By type is meant a specimen or sample possessing in a marked degree the leading characteristics of a class of things. A classification by type, then, consists in choosing those specimens and arranging them by degrees from the most perfect species to the most simple. According to this system an object presented to us will be easily classified by comparison with the respective type.

The utility of such classification is obvious. It affords a means of illustrating objects in the class-room, in a book or a museum. Besides, a series of types well selected shows to the eye the outer characteristics of the species or group, and so helps the memory. Yet a mere classification by type is superficial, deceptive and insufficient for scientific purposes. What we want is unity in each group, not only in name and appearance, but in concept and meaning. Now, no two individuals are really equal ; hence the necessity of taking essential characteristics, which by abstraction are the same and capable of one definition.

Classification by Kinds. Natural kinds are forms of things, mineral, vegetable or animal, whose definite characters are supposed to have been fixed from the time of their origin. In other words, they came into existence by an act of creation as we see them in the order of nature. Classification by kinds, therefore, is the orderly arrangement of the objects of one science into groups, from the most perfect and complex to the simplest forms. The result is a classification by series. Thus in Botany the classification begins by the most particularized plants called *species*; these are grouped under *genera*; genera under *orders* or *families*, etc., till we arrive at two primary groups, the *flowering* and the *flowerless* plants, which constitute the Vegetable Kingdom. Observe that each characteristic group admits of accidental differences called *varieties* which are due to peculiar conditions of soil and climate prevailing in the locality, or to artificial devices. Such conditions, however, do not interfere with the specific characteristics, which are supposed to be fixed in the course of nature.

In Zoology the whole of animal life is likewise arranged in order according to degrees of perfection. Material substances obtain a similar classification in Chemistry. Examples:

1. Oxygen has been found in the soil, in all rocks and minerals. More than half the weight of our whole earth consists of oxygen. Hydrogen has been discovered in all acids, nitric acid, sulphuric acid, etc. Many material substances, therefore, will come under one or the other of these simple elements, forming a series of compounds according to the constituent elements.

2. Animals are classed into vertebrates and invertebrates; the group vertebrates into mammals, birds, reptiles, amphibious, fishes, etc.

3. Bentham's *British Flora* is a classification of
Flowering plants into
Compound flowers—Not-compound flowers.
Not-compound into
Those of one seed—more than one seed.
The latter into
Those of perianth single—perianth double.
Perianth double into
Corolla of one piece—not of one piece.
The latter into
Those of ovary free—ovary not-free.

Observe that each group or class-name will embrace those individuals in which the connotation is verified and by which they are all the same, although varying in other respects.

Classification by Evolution. The classification according to natural kinds has been modified in modern times by the theory of Evolution.

The idea of Evolution is that the present species of living beings, plants for instance, have developed in the course of ages from one or a few simple forms of life, passing through gradual stages which are accounted for by the so-called law of "Natural Selection." It is supposed that some descendants of a common parent survived and thrived under favourable conditions of soil and climate, while others decayed and died.

This supposition, the defenders of Evolution say, is supported by certain facts, namely, the slight variations noticed in the species of to-day. Their reasoning is like this : Those variations have occurred during countless ages, and must have produced an effect. This must have been at first a difference of degree only between the descendants and their ancestors, but this by accumulation through successive generations became a specific difference, and the new species again by further variations became a genus, and so on. The final result was a set of relations, like those existing among the members of a family. The plants, for instance, belonging to a common species descended in a way from those that appertain to the genus, and these in turn reckon as their ancestors the plants included in a higher class.

Evolution is a hypothesis that may be true or not, or true only in part ; but many scientists admit it, after Darwin, Huxley and Spencer, as a possible way of explaining the formation of species. Furthermore, the hypothesis accounts for many phenomena observed in plant life, for example, the natural varieties of individuals under different conditions, likewise the particular results obtained by artificial horticulture. To the objection that our experience can show no instance of a new species formed under our eyes, they answer that specific variations evolve so slowly that it takes an immense time before they are realised.

Classification, as modified by evolution, differs from that based on natural kinds in two ways :

(*a*) The important characteristics according to evolution are the supposed constant marks of hereditary descent, which establish a relationship between a large number of living forms, like that between the members of a family ; while important characteristics in natural kinds are those which imply greater and deep similarity.

(*b*) In evolution an attempt is made to discover reasons or supposed laws which explain such great variety of existing species ; while in natural kinds the scientist pays attention to correlations of properties recognised in the individuals of the vegetable and animal kingdoms.

As to the value of either way of classifying living forms—a task surrounded with difficulties, Jevons makes a remark that holds good to this day: "There are causes of likeness apart from hereditary relationships, and *we must not attribute exclusive excellence to any one method of classification.*"

5. The Guiding Principles of Classification. Two principles will serve as a guide to work out both artificial and natural classifications, particularly the latter which aims at the knowledge of things.

Principle I. Choose mentally as the centre or nucleus of your classification the attribute or attributes that are *most important*, and which may run through all the subordinate classes up to the highest genus. The result will be that looking at your work, once completed, from the highest genus down to the lowest species, you discover your fundamental attribute binding them all in a descending line.

What is meant by *most important* attributes is not easy to say. In the words of Mill it means "those which contribute most, either by themselves or by their effects, to render the things (in any class) like one another, and unlike other things (of other classes); which give to the class composed of them the most marked individuality; which fill, as it were, the largest space in their existence, and would most impress the attention of a spectator who knew all their properties, but was not specially interested in any."[1] Examples:

1. The simple elements, namely, those that cannot be further decomposed, are classified in Chemistry by a good fundamental attribute, the atomic number, which runs through the whole series.

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[1] Mill, *System of Logic*, Vol. II., p. 271.

2. The classification of clouds has been accomplished of late by a most important attribute, viz. their average altitude.

3. In Bentham's *British Flora* the chosen character is the structure of the flower, not its colour or any other superficial element.

4. Regarding artificial classification, vehicles (for instance), may be most conveniently classified by their motive-power as this feature is most important. Again, in a Trade Directory, the multitude of trades may be arranged by alphabetic order of names for the purpose of finding their location or address. A better way would be according to the need for which society or individuals demand their existence. Some trades look to the well-being of society, others meet only individual needs.

Principle II. When passing from the lower species to the higher genus, and to co-ordinate species select as difference an attribute that is *most important.* The result will be that looking at your classification, once completed, groups most distant will be most unlike, and the distance of one group from another in the scheme will be an indication of their dissimilarity.

Observe that those principles correspond to or resemble the lineal and collateral relationship of a family, thus helping to lay before us the relations of things. They also point out the degree of relation according to the order established in nature. Examples :

1. The main groups of clouds differ in altitude, and the subclasses are distinguished by their peculiar forms.

2. The classification of books according to the decimal system is based on ten fundamental groups. Then each one branches out by a definite character ; for instance, the History group embraces the various kinds of History, that of Philosophy the kinds of Philosophy, etc.

We have suggested, from the stand-point of Logic, the principles leading to an ideal classification. In practice, that is to say, face to face with the individual things of nature, the work is very difficult. To realise this, let the reader try for himself any classification whatever. The difficulty grows and is at times insurmountable when attempting natural classifications. For the problem of classifying extends to various sciences and demands an insight into the constitution, character, relations and origin of the natural objects. Yet natural things are graded without a visible break ; *Natura non facit saltum.* One class of things shades imperceptibly into another, so that it is difficult for us to distinguish individuals lying on a borderland.

It is true that scientists go by points of resemblance and difference which appear on the surface of things. Yet is it safe to judge by appearance ?

True, we find resemblance, similarity, affinity in nature. But exterior signs are not all of the same tone or equally far-reaching ; they require interpretation. Hence the labour of studying that similarity which is profound, making it reveal what lies behind, the very constitution of things. Once an essential attribute is discovered, then and then only do we possess the clue to discovering the nature of other similar things. Examples :

For a long time the science of Meteorology tried to improve its classification of clouds. Revised schemes had been promulgated from time to time, till the International Meteorological Conference held at Munich in 1891, where a committee was appointed finally to settle the problem. An atlas was drawn up showing the typical forms of clouds ; instruments were devised, and the work was carried out by the co-operation of fourteen Observatories scattered over the globe. Here is the result of so many experts on the matter.

International classification of Clouds :

(*a*) Separate or globular masses (most frequently seen in dry weather).

(*b*) Forms which are widely extended, or completely cover the sky (in wet weather).

A. **Upper clouds,** average altitude 9000 metres.
    (*a*) 1. Cirrus.
    (*b*) 2. Cirro-Stratus.

B. **Intermediate clouds,** between 3000 metres and 7000 metres.
    (*a*) 3. Cirro-Cumulus.
    4. Alto-Cumulus.
    (*b*) 5. Alto-Stratus.

C. **Lower clouds,** 2000 metres.
    (*a*) 6. Strato-Cumulus.
    (*b*) 7. Nimbus.

D. **Clouds of diurnal ascending currents.**
    (*a*) 8. Cumulus, apex 1800 metres, base 1400 metres.
    (*b*) 9. Cumulo-Nimbus, apex 3000 to 8000 metres, base 1400 metres.

E. **High fogs** under 1000 metres.
    10. Stratus.

Each group became fully known as to essential and accidental characteristics. See, for instance, the following explanation :

Cirrus (Ci).—Detached clouds, delicate, and fibrous-looking, taking the form of feathers, generally of a white colour, sometimes arranged in belts which cross a portion of the sky in great circles and by an effect of perspective, converge towards one or two points of the horizon.[1]

Similar examples of hard work and long time spent in classification may be seen in the history of various sciences. Needless to add that most classifications are subject to revision with the advancement of knowledge.

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[1] The English text of this classification is taken from *The Encyclopædia Britannica,* 11th ed., Art. Cloud.

6. Division and Classification. Comparing now what has been said of Division in Deduction with the process of Classification, it is plain that both are alike and in a certain sense coincide. If we look only at the matter divided and at the groups into which it is divided, the field covered is the same.

The process, however, that we follow in each is very different. Division proceeds downwards. It begins with the genus or a wide class, and descends to the narrower classes and then down to the individuals. Classification works the other way about. It begins with the individuals scattered in nature, and from them ascends to a class common to them, and from that class up to a wider one, till we reach the widest possible in each science.

Again, in Division we use our previous knowledge ; and according to this we divide a whole extension into groups of less and less extension at each step. An easy process particularly suitable for instruction.

In Classification we depend on experience, by which we gain gradually our knowledge of things. The process is suitable for fresh investigation, in which the scientist has to break new ground in search of the unknown.

Observe that the rules given to test a good division apply also to classification when this has been completed, but not in the process of making. Those rules are rather formal, and work well in arranging our previous knowledge.

The names used in division are only species and genera related to one another at each step. In classification the name species is applied only to the lowest, and genus to the wider class immediately above the species. Each class above receives a distinct name. For instance those employed in Botany are : Individual, Variety, Species, Genus, Order or Family, Class, Sub-Kingdom.

7. Nomenclature and Terminology. Let us remember the purpose of Classification, namely, to build up the connotation of names and to establish their relation. So far we have accomplished the classification of things according to their characteristics, and have tried to discover the order existing in nature. The next problem is the giving of suitable names.

Observe here the wisdom of science. Not every name taken from current language will do ; for language changes the meaning of names, now using the same name for different things, now forcing them to express metaphorical meanings. Science needs to be precise and exact. A name in science should mean one thing, and always the same thing. Hence the necessity of devising names taken from dead classical languages. A name thus selected will convey as much meaning as possible, precisely, and is not liable to corruption.

To that end the ancient scientists appropriated names more or less in common use. For instance, the names, Sphere, Cone, Cylinder, were reserved for Geometry ; other names like Zenith, Nadir, Horizon, Latitude, Longitude became Astronomical, etc. The appropriation consists in making those names bear the meaning of a definite attribute in reference to a theory unfamiliar to the common people. As science developed in modern times, names of that kind were extensively devised and reduced to a system of classification.

Nomenclature is a system of names adapted to the various groups or classes of a natural classification.

Terminology is a collection of names fit to describe the qualities or the parts of an object. The purpose in view is threefold :

(*a*) To preserve the classification ; that is to say, to keep it always the same. As the names are unfamiliar there is no danger of their being turned to various meanings.

(*b*) To generalise the classification, for those names belong to no particular language but the scientific. Whoever learns a science must learn it under those names.

(*c*) To express the classification, conveying at once the nature of things and their relation. This last condition requires some explanation.

In Geometry the names *Tetragon*, *Tetrahedron*, *Pentagon*, *Pentahedron* express the kind of figure and the number of angles.

In Botany flowering plants are classed into *Dicotyledons* and *Monocotyledons*, meaning respectively plants with two *cotyledons* or seed-leaves and plants with one *cotyledon* or seed-leaf.

The science of Chemistry has adopted certain terminations of the words to increase the meaning. Thus speaking of *Sulphuric* and *Sulphurous* acids, or *Sulphates*, *Sulphites* and *Sulphides* of metals, indicates at once the constitution of the substance and its place in the system.

There is no need to multiply examples, which any one can find in the respective sciences. What makes to our purpose is that a good Nomenclature conveys in an exact manner the connotation of names and thei various relations. It helps to retain the science in our memory, and at the same time aids the understanding. Each name is equivalent to a condensed proposition telling what a thing is. In fact a classificatory science is wholly contained in its classification. We may remark that each name is a definition of the object signified—a logical definition if the attribute referred to is a specific difference ; if not, the name is equivalent to a description.

In regard to Terminology, the names employed are meant to suggest the external appearance. The science of Botany possesses a large terminology to express the parts and varieties of roots, stems, leaves, flowers and fruits proper to each plant. The flower, for example, is made up of *Calyx* with its *Sepals*, *Corolla* with its *Petals*, *Stamens* and *Pistils*.

8. **Naming in general.** Looking now in general at the formation of common names, it may be said that in every language the process of naming things has been similar to that of scientific classification.

Philological analysis demonstrates that most names are derived, as to sound and meaning, from simple forms called roots. These roots were meant originally to express simple concepts, but gradually passed on to signify other things. By further analysis and synthesis, (that is to say by various combinations) they gradually became complex and their meanings extended. Take as a familiar example the words print, printing, printer, printing-machine, print-shop, printed paper.

Again, the parts of speech are traced to a few most fundamental notions called predicaments, which reveal a philosophical analysis of things as they are in themselves. Their meaning has afterwards been extended by combination. Compare the following words : substance substantive, substantiate, substantial aid.

There is a considerable difference, however, between the formation of scientific names and that of common names. Both kinds possess connotation and denotation; but the meaning attached to names in current language is not always precise, and the relation of names among themselves is not exact and systematic. This is only to be expected ; for they have often been formed in a loose manner by ordinary people, as use or necessity required. Hence our special difficulty in reducing them to system by the process of logical division and definition.

As a final remark, observe how the process of naming agrees with the treatise on names at the beginning of Deduction. It appears that beginning and end meet together as completing a circle of knowledge. In fact it is true that definition and division are both the beginning and the end of knowledge—the beginning from the point of view of imparting knowledge ; the end looking at the process followed in formation of names. Generally speaking, Deduction begins where Induction ends.[1]

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[1] For further explanation on Classification and its bearing on Definition, Division and Predicables, see Jevons, *The Principles of Science*, Chap. XXX., Carveth Read, *Logic Deductive and Inductive*, Chapters XXI., XXII., XXIII.

CHAPTER XVII

FALLACIES

1. Sources of Fallacies and Errors. It has been our purpose in the preceding chapters to explain the process of correct thinking, both in deductive and inductive reasoning. That process shows the way we ought to think. But people do not always think as they ought ; and the crooked ways of using reason may be traced to several common sources.

In the first place we should call attention to the utmost importance of holding sound general principles. " One of the most fertile sources of errors in the history of thought has been the assumption that certain general propositions, which involve the conclusions drawn, are axiomatic."[1] In fact a single wrong principle can spoil a whole system of doctrine. Take, as familiar instances, the following two propositions : " Instruction is highly beneficial to society," and " Instruction makes people seditious and discontented," and consider the consequences one commits oneself to by holding one or the other.

The limitation of our knowledge is another source of error and fallacy. It often happens that facts are wrongly analysed for lack of previous knowledge. And likewise one fails to grasp the right meaning of a term or the sense of a proposition, because of insufficient mental preparation. Names like Pleasure, Possibility, Right, Obligation, and a hundred more, imply meanings, the distinction of which requires a great deal of study.[1] To say it briefly in the words of Bacon : " Men imagine that their minds have the command of language, but it often happens that language bears rule over their mind."

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[1] J. Welton, *Groundwork of Logic*, p. 119. This writer illustrates with striking examples the evil consequences of false axioms.

Distraction is another source. At times we look at an inference and declare it right, when in reality there is no inference, as we shall discover on closer consideration. The saying : " Even Homer sometimes nods " is here to our purpose. On occasions a subjective disposition inclines us to call certain what is merely probable. Prejudice and personal interest have a great deal to do with the formation of our beliefs.

Errors and fallacies, whatever the sources may be, are so frequent in our discourse that we need not lay stress on the importance of detecting them.

2. What a Fallacy is. Errors and fallacies issue in like results ; an error deceives, and a fallacy leads us to error. The two, however, are distinct notions easily understood.

Error means simply departure from truth, either by assuming as true premises which are false, or else by a blunder in reasoning, so as to get a false conclusion out of true premises. Error implies good faith, but cocksureness based on ignorance or prejudice.

Fallacy means deception, active or passive. So far as the foregoing errors are committed in good faith, there is *passive* deception of the propounder or reasoner under a semblance of truth. So far as the same errors tend to deceive the hearer under semblance of truth, there is *active* deception. One might say that all error is fallacy in that it tends to deceive the unwary. But we generally apply the term to such errors as are easily fallen into by the unwary propounder, or easily deceive the unwary listener, and bear a surface plausibility which requires vigilance and keen criticism to detect. If an error is a simple blunder which is easily seen through, we hardly call it a fallacy. A fallacy always means something hidden, tricky ; a trap, a pitfall which one can fall into easily, and get out of with some difficulty. We might say that error is the genus, and fallacy is the species, i.e. error calculated to take people in.

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[1] See Whately, *Elements of Logic*. This source of fallacy is fully and skilfully analysed by this author in Book III.

A fallacy, says Whately, is " any unsound mode of arguing, which appears to demand our conviction, and to be decisive of the question in hand, when in fairness it is not." Briefly, a process of thought which appears valid and is not. For the purpose of Logic it is immaterial whether the fallacy be intentional or unintentional. The speaker's dishonesty, however, has introduced a distinction of fallacies, since the time of the Sophists ; that between Paralogism and Sophism. The former is simply a form of argument not in accordance with the rules. The latter adds the idea of an insidious argumentation, by which one intends to deceive ; sophistry is pretended wisdom.

3. Division of Fallacies. Logicians do not agree as to the manner of grouping fallacies. The fault lies in the fallacies, which are capable of taking varied subtle and elusive forms. Hence it happens that one and the same fallacy may be included under different heads.

One way of avoiding the difficulty is to distribute them according to the chapters of Logic. Welton writes: "The most convenient classification of fallacies seems to us to be one based on the logical principle violated in each case. We shall thus consider in order fallacies incident to each of the divisions of logical doctrine."[1] The order is: Fallacies incident to Conception, to Judgment, to Immediate Inference, to Deductive Inference, to Inductive Inference, to Method. We shall arrange them under certain unities in order to aid memory:

(*a*) **Formal fallacies**, consisting in the violation of logical rules, so that the conclusion does not follow from the premises.

(*b*) **Material fallacies**, misleading on account of the wrong application of the matter itself.

A. **Purely formal**, or those against the rules:

(*a*) 1. Illicit forms of immediate inference by opposition and by eduction.
2. Illicit process of major and minor terms.
3. Undistributed middle term.
4. Two negative, two particular premises.
5. Four terms in the syllogism.
6. Wrong moods in hypothetical syllogisms.
7. Wrong moods in disjunctive syllogisms, etc.

B. **Semi-formal fallacies**, or terms used in a double sense:

(*b*) 1. Equivocation, Amphibology.
2. Composition and Division and vice versa.
3. Fallacy of Accident.

C. **Begging the question** in various forms:

(*b*) 1. Assuming the conclusion.
2. Arguing in a Circle.
3. Unduly assumed premises.

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[1] See J. Welton, *Manual of Logic*, Vol. II., p. 235. This author gives an account of Aristotle's, Whately's and Mill's classifications besides his own.

D. **Evading the question** in various ways :

(*b*) 1. Irrelevant conclusion (Appeal to the individual. Appeal to the people. Appeal to a man's sense of shame).
  2. Shifting the ground.
  3. Fallacy of many questions.
  4. Fallacy of objections.

E. **Inductive fallacies,** liable to occur in the three steps of the process :

(*b*) 1. Fallacy of simple inspection.
  2. False cause.
  3. False generalisation.
  4. Proving too much.

Here we shall pass over the purely formal fallacies, as sufficiently dealt with in treating of arguments.

4. **The Semi-formal Fallacies** are at once formal and material. Formal because a certain rule is violated, material because this violation is only discovered by taking account of the meaning of the proposition.

Equivocation, from the Latin *Equivocatio*, means using the same word in two senses. Examples :

1. Animal is a genus, therefore your horse, being an animal, is a genus.—Here the name animal is first used as predicable or reflex notion, and then as a common name or direct notion. Hence the fallacy of four terms.

2. You are not what I am, therefore you cannot be a man, for I am a man.—Again the same fallacy, since 'I' in the major premise is taken individually, while in the minor it is understood specifically.

3. Happiness is the end of life, therefore pleasure, being happiness, is our purpose in life.—A distinction between rational and irrational pleasure is here overlooked. Only rational pleasure coincides with happiness ; whereas the conclusion means that any kind of pleasure is to be looked upon as the purpose of life.

4. What happens every day is very probable; things improbable happen every day; therefore things improbable are very probable.—The expression, 'happens every day' means continuity of happening in the major premise, while in the minor it implies happening here and there, under various circumstances.

5. No one who has not enough is rich; a miser has not enough; therefore a miser is not rich.—Notice the two meanings, not enough in possession, and not enough in desire or ambition.

6. Whoever intentionally kills another should suffer death; a soldier intentionally kills another; therefore he should suffer death.—There are two intentions in killing, one malicious, the other out of duty or justice.

Another name for this fallacy is *Amphibology*. Expressions in common language like these: "Not at home; I do not know," are cases of double meaning. One is what they materially mean, the other what they intentionally mean.

Composition and Division, or in the old phrase, *A sensu composito ad divisum*, and vice versa. Two things are taken together in one place and separately in another—or the other way about. The predicate of the proposition naturally suggests the true sense. This fallacy is closely allied to the previous one. Examples:

1. The works of Shakespeare cannot be read in one day; therefore *Hamlet*, which is a work of Shakespeare, cannot be read in one day.—There is here a collective and a distributive use of "works of Shakespeare."

2. Animal food may be dispensed with, and vegetable food also. But all food is either animal or vegetable; therefore, all food may be dispensed with.—"Dispensed with" in the major premise is understood separately, that is, "if there be another kind of food"; while in the conclusion it is meant absolutely or altogether

3. Whoever must go or stay is not free ; but you must either go or stay ; hence you are not free.—Two meanings of the phrase " go or stay " ; one conjointly in the major premise, the other alternative in the minor.

This fallacy is plain enough. Yet it is remarkable how easily people pass judgment on an individual and then extend it to a community, or vice versa. It is not uncommon to say : " This nation is advanced in education," and then infer that every individual of that nation is highly educated. A member of the board is trustworthy, hence the whole board is likewise so. A Christian has done wrong, consequently the Christian community is also wrong. Fallacies of this kind may be referred to the type of division and composition, or vice versa, composition and division.

The Fallacy of Accident consists in confusing what is primary and what is secondary in an object. It is almost the same as that called by the ancients *a dicto secundum quid ad dictum simpliciter* and vice versa, which means passing from what is true of a certain quality, condition or circumstance to a statement that makes it true in other respects or in a general way. Examples :

1. One thinks to himself : That man dresses as a European, therefore he is not a Mahomedan.—A fallacy from an accident to the nationality of the person.

2. People often reason in this way : A certain man, say Mr. Edison, is eminent in scientific research ; therefore his opinions on other matters, for instance, religion or the immortality of the soul, must carry great weight, for he is a learned man.—The inference is fallacious, since what is true of his scientific knowledge is extended to other kinds of knowledge which he may not possess.

3. One may argue in a serious matter thus : Man is free to dispose of his own actions, but a binding law violates that

freedom ; therefore binding laws are not justifiable, being against individual rights.—The argument proceeds from what is true under conditions, to an absolute sense. Every man is free on condition that there is no law above him. A binding law violates freedom in an absolute sense, which of course does not exist. Hence the conclusion is totally wrong, as no right is violated by a just law.

4. "The opposite form of the fallacy," says R. Clarke, "which argues from something generally true and undeniable to the same when some special condition is introduced, is also a very frequent and often a very pernicious one. The teetotaler who refuses to give wine to the sick, even when the doctor orders it, on the ground that it is dangerous to take stimulants ; or the parent who will not correct his pilfering child on the plea that it is cruel to beat children ; or the theologian who condemns Abraham's intention to sacrifice Isaac, on the ground that murder is always unjustifiable, are all guilty of arguing *a dicto simpliciter ad dictum secundum quid*. The whole class of narrow-minded people who get some idea or principle into their heads and apply it, irrespective of circumstances, are all sophists, though they know it not." [1]

5. Material Fallacy Explained. What is precisely wrong in the procedure of a material fallacy cannot be said in one word. The fallacy does not consist in the violation of the rules of formal reasoning, but lies in the fact that the premises used are not a ground for the conclusion to be proved, so that we offer as proof what is not a proof. An argument of this kind may be such that the inference is correctly drawn, and yet involves a fallacy. What then is unsound in it ?

Let us use a comparison : A pair of reading-glasses may be perfect of their kind, and yet a man may not be able to read with them. This may be for one of three reasons: either because the man has never learnt how to read, or because the glasses are unsuited to his eyes, or finally because the man is blind. In a similar manner all material fallacies may be reduced to three general defects, namely:

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[1] Richard F. Clarke, S.J., *Logic*, pp. 447-8. His account of fallacies is clear and interesting.

(*a*) Deficiency at the starting-point by assuming premises that ought not to have been assumed. This resembles the man who tries to read with glasses, but has never learnt to read.

(*b*) Proving a conclusion that is not required, or is unsuited to our purpose—or in other words, proving something else that may be mistaken for the true point. This is the case of the man who uses glasses unsuited to his eyes.

(*c*) Employing without sufficient evidence, that is blindly, facts or theories to work out an inductive generalisation. And this is the third case of our comparison.

We pass on to exemplify these three heads of material fallacies. All the various kinds usually mentioned may be included under one or other of these three.

6. **Begging the Question,** or in the old phrase *Petitio principii*, means that one assumes the principle or ground; that is to say, a conclusion is proved by a premise taken for granted as needed for that conclusion, and surreptitiously assumed; or else a conclusion is proved by a premise which in part or in whole depends upon the conclusion. Original writers, especially in abstract thinking, fall occasionally into this fallacy. Examples:

1. If any argument whatever does not go beyond the premises, it teaches nothing new; and if it goes beyond the premises, it is invalid. But an argument either goes, or does not go beyond the premises; therefore, any argument

whatever is either invalid or teaches nothing new; consequently it is useless to argue.—The fallacy lies hidden in the first hypothetical proposition, where surreptitiously it is taken for granted that an argument ought to teach something beyond the premises taken together. A right conclusion cannot go beyond the premises taken together, but it does go further than each of them, as something new and not grasped till in the course of argument each premise is seen through and accepted. In other words, the connection between the premises shows the truth of the conclusion. That is the merit of an argument; not any advance upon the premises taken together, as the fallacy implies.

2. It is fated that you will either die or not die of the present disease, for the two alternatives are contradictory, one of which is true and the other false. Therefore, it is useless to call in a doctor who cannot reverse that decision. —The fallacy consists in the surreptitious assumption that the doctor cannot influence the truth of the one alternative rather than the other. This is stated in the conclusion and presupposed in the premises. There is consistency between the premises and the conclusion in the supposition that one alternative is true and the other false independently of the means employed; but precisely that supposition is wrongly assumed and stated in the conclusion.

Arguing in a Circle is another type of begging the question, known as *Circulus vitiosus* or "vicious Circle." This occurs in a train of reasoning by assuming as premise a proposition to prove a conclusion, when this conclusion was previously employed to establish that very same premise. As the reasoner makes no advance from something known to something else, but returns to the same thing, he is said to move in a circle. Examples:

1. J. S. Mill attempted to prove the uniformity of nature from experience; and then, assuming that uniformity as a universal principle, deduces by the methods of Induction the various uniformities that exist in experience.

He says: "Whatever be the most proper mode of expressing it, the proposition that the course of nature is uniform is the fundamental principle, or general axiom, of Induction. It would yet be a great error to offer this large generalization as any explanation of the inductive process. On the contrary, I hold it to be itself an instance of Induction, and Induction by no means of the most obvious kind."[1] This idea is not a passing one in Mill's Logic, it reveals itself in several places.

Let us call to mind what has been said elsewhere; viz. that the uniformity of nature cannot be proved from experience. But if it were, we could not assume the same to establish its existence in the course of experience. The fact is that the principle of the uniformity of nature is presupposed in Induction. The fallacy would be avoided by proving the existence of that uniformity by a certain kind of knowledge other than experience.

2. Descartes in one place of his *Meditations* maintains that a clear and distinct idea must be true, because God, being the author of those ideas, cannot deceive us; and in another place he tries to prove that God exists, because of the clear and distinct idea we have of Him.

3. In like manner, Plato proves in his *Phaedo* the immortality of the soul by its simplicity, and in his *Republic* he proves the simplicity of the soul from its immortality.

Unduly Assumed Premises. This is a fallacy regarding the starting-point, which may be classified under begging the question. It consists, as the name shows, in taking as premises general propositions which are untrue, or not universally true. For instance, if one assumes with Descartes as a true definition of matter, that it is extended substance, and as such can neither move itself nor another, he will be able to deny the Newtonian theory of gravitation with a correct formal argument. This follows from a single false notion assumed as true. It is more common to take for granted two or more suppositions, or false premises. Examples :

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[1] Mill. *System of Logic*, Vol. I., p. 342.

1. A defender of evolution may argue thus : Natural selection, after the manner of artificial selection as used in horticulture, brings about small variations in certain individuals of the vegetable and animal kingdom ; a phenomenon which Herbert Spencer calls the " Survival of the Fittest." These variations, being inherited, accumulate with further variations till a sufficient amount is reached to constitute a new species. The efficacy, therefore, of natural selection is the sole originator of species.—In this argument three assumptions are made and taken for granted, namely, that natural selection is the cause of variation ; that such variations are inherited ; and that these variations reach the point of a new species. As none of these assumptions are known to be true, the argument is a big fallacy of unduly assumed premises to establish the origin of species.

2. Good government is conducted either by reason or by force. If by reason alone, many will not listen to it ; if by force alone, people will revolt. Therefore good government is impossible.—There is a third alternative in the disjunctive premise, namely, a due admixture of both reason and force ; and this precisely makes a good government.

3. The world is a combination of elements that must originally have come into existence either by accident or by a blind necessity. The first cannot be maintained ; consequently, the latter is true.—A third alternative is left out, that it came about as an effect of a free and omnipotent cause.

4. Logic deals either with the objects of thought or with the forms of thought. But Logic has nothing to do with the objects of knowledge, which appertain to the various sciences ; therefore, it follows that the proper object of Logic is nothing else but the study of the pure forms of the understanding, that is to say, their consistency.[1]—It

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[1] Ueberweg, *System of Logic*, p. 533, attributes this fallacy to Kant.

is plain that Logic has to do with something else, namely, the agreement of thought with the objects we think about. To avoid ideal contradiction is one thing and to avoid real contradiction is another, and both are under the domain of Logic The former means consistency of thought, the latter is formal truth.

5 That the death-penalty is unjust may be proved in directly thus: The death-penalty is said to be just inasmuch as it prevents wrongdoing. If so, the same penalty will also restrain people from small thefts and other like trespasses. Therefore, the death-penalty will be justly imposed for every transgression of the law, which is absurd. —The reason why the death-penalty is just is not that it prevents wrongdoing in general, but because it is a just punishment for enormous crimes. Due proportion is what makes that penalty just.

7. Does the Syllogism Beg the Question? Observe that in a syllogism the premises together contain the conclusion; and this is true of all legitimate arguments. The question is whether the conclusion is presupposed for the validity and truth of the premises.[1]

Carveth Read says: "If *all* the facts of the major premise of any syllogism have been examined, the syllogism is needless; and if *some* of them have not been examined, it is a *petitio principii.* But either all have been examined, or some have not. Therefore, the syllogism is either useless or fallacious.—A way of escape from this dilemma is provided by distinguishing between the formal and material aspects of the syllogism considered as a means of proof. It begs the question formally but not materially." [2]

To place the syllogism on its proper merit, we shall put the matter in a somewhat different way. The universal premise of a syllogism—for there must be in a syllogism a universal premise which implicitly contains the conclusion—may be analytical or synthetical. If analytical, it does not depend for its universality upon the individuals examined. A synthetical proposition depends upon observation of individuals; but scientific induction arrives at a universal proposition *materially* true by discovering a natural connection between subject and predicate, in such a manner (as explained before) that the number of instances examined is of no importance to establish universality. A third case is that of a universal proposition *formally* true, or established by complete enumeration of instances. Using such a proposition in a syllogism, some logicians say, is useless, and besides is begging the conclusion. For the conclusion has been required to build up the universal premise; the word all includes the particulars of the conclusion.

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[1] See what has been said in the Chapter on Arguments, No. 7.

[2] Carveth Read, *Logic*, pp. 163-4.

We prefer to say that in this case—the only one in question,—the syllogism is neither useless nor fallacious; because the syllogistic evidence does not consist in one premise only, but in the connection of major and minor. It is true that the syllogism is useless for one who built up the syllogism and positively used the conclusion to establish the premises. But for one who receives it as information, without knowledge of the data of the premises, the syllogism stands as a proof and begs no question. He comes to know the conclusion legitimately from the knowledge of the premises, and in no way vice versa. The syllogism is a proof *to him.* "If I learn," says Mellone, "that the vessel XY was lost at sea with all on board, and learn subsequently, or by some other means, that my friend AB was a passenger on that vessel, then there is no doubt that the conclusion is something new although the major states a mere collective fact, which (for those who know, but not for me) already contains the conclusion." [1]

8. **Evading the Question.** Another general type of material fallacy consists in proving or disproving what is not the matter in question. The speaker really ignores the point at issue; hence the traditional name, *Ignoratio elenchi*. Examples:

1. The practice of Religion is optional to everybody. Proof: There are people who practise Religion, whose conduct is not better by it. But a practice which does not improve conduct cannot be obligatory, and can only be optional. Therefore, the practice of Religion is optional to everybody.—The word optional is opposed to obligatory. The proof should be to the effect that the practice of Religion is not one of the duties of men. To show that some people do not profit by it, is irrelevant.

2. The question is raised, whether reduction of salaries is necessary in order to save a certain institution, and a member of the Board says: On no account we should reduce salaries; because we must not do evil in order to derive benefit, and such reduction of salaries does harm to our employees.—The proof is beside the mark. The argument should consist in showing, by appealing to facts, that the institution is or is not able to continue without reduction of salaries. The harm pointed out follows accidentally to some extent, but this is not aimed at by the proposal, but rather the contrary; a reduced salary is better than none.

3. "The skilful barrister will often seek to draw off the attention of the jury from the real point at issue, viz. the guilt or innocence of the prisoner, by a pathetic description of the havoc that will be wrought in his home if he is con-

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[1] Mellone, *An Introductory Text-Book of Logic*, pp. 231-2. The reader will find a very interesting account of the utility of the syllogism in Minto's *Logic*, pp. 209-14.

victed, or by seeking to create an unfair prejudice against prosecutor or witnesses. The host who seeks to enhance his guests' appreciation of his wine by letting him know what it cost him, really ignores the point at issue, which is, not whether the wine is expensive, but whether it is good "[1]

Arguing beside the point debated is at times rightly used and brings home persuasion in the following ways :

Appeal to the Individual or *Argumentum ad hominem.* In this case the disputant does not defend the proposition attacked, but merely shows that his opponent is not the man to contradict the statement. Examples :

1. Someone attacks Religion on the ground of being superstitious. I reply that he is superstitious himself, for he refused to sit at table because the number of persons was thirteen, and he will never set out on a journey on Friday, which for him is a day of bad omen.
2. The Pharisees accused Christ of curing on the Sabbath-day. He answered them : " What man is there among you, that hath one sheep ; and if the same fall into a pit on the Sabbath-day, will not take hold on it and lift it up ? "

Appeal to the People or *Argumentum ad populum.* The argument is directed to arouse the feelings or prejudices of the audience, thereby to discredit an established doctrine. Examples :

1. A socialistic leader persuades the working classes that the right to property is unjust and fictitious, appealing to their passions. The argument is a pathetic description of their suffering, trying to show that they produce, while the rich enjoy the products of their hard labour.—This is no argument against the right of ownership.
2. Similarly a demagogue may declaim against authority thus : Are you not free-born citizens ? And what is freedom without the exercise of it ? And what exercise is left to us if another imposes a law not made by us or our repre-

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[1] **Clarke, S.J.,** *Logic*, **pp. 448-9.**

sentatives ? Hence autocratic government is unjust, and democracy alone deserves the name of government.—Democracy is only one kind of government. The argument does not prove that monarchy is wrong.

An Appeal to a Man's Sense of Shame. The Latin name is *Argumentum ad verecundiam.* It means accusing one of lack of modesty in over-estimating his talents. Examples :

1. One gives reasons for his doubts as to the truth of the theory of Evolution. His opponent says : How can a young man, without sufficient experience, dare to put himself up against men of science such as Darwin, who spent a lifetime in investigating the origin of man ?—The argument is irrelevant and does not contradict a single reason. Yet it invalidates the external authority of the opponent.
2. Likewise, if one of the community denounces as useless a traditional celebration, or a certain manner of cultivating the fields, etc., a more ancient member replies : Such is the custom. Do you know better than our forefathers for generations ?—The answer proves nothing against the reasons offered, yet it is likely to persuade the hearers in favour of the established tradition.

Shifting the ground means passing on to a proposition different from that to be established. Examples :

1. An agent is accused of inefficiency in his work, and his defence is as follows : The results obtained in this same office in previous years were not much better than what we have at present. Nobody has been blamed in the past, why should I be ? —The reply avoids the personal question by diverting attention to other people.
2. A father to his son : Have you taken those sweets ? The son : My brother must have told you.

The Fallacy of Many Questions. This means that we put a question with an assumption ; that is, an insinuation is thrown in, by way of assumption, and a man by answering the question unwarily grants the assumption. Examples :

1. A judge to an accused : Were you drunk or sober *when you were beating your wife ?* Or again, where did you hide the goods *you stole last night ?*—The accused, by answering the question asked, falls into the trap of seeming to acknowledge the thing *assumed.*

2. Let us consult the past. Were not Socrates, Caesar Augustus, Francis Xavier, Emmanuel Kant, noble characters wholly devoted to the welfare of humanity ?—These men have been great in different ways. To take them together is a confusion that leads to anything.

The Fallacy of objections, as Whately says, consists in showing that there are objections against some plan, theory, or system, and hence inferring that it should be rejected ; when that which *ought* to have been proved is, that there are *more* or *stronger* objections against the accepting than the rejecting of it. By this fallacy anti-innovators are accustomed to obstruct indiscriminately the best reforms and alterations ; for there is no plan against which objections may not be urged. Take an instance. Capital punishment has objections, Transportation of criminals has objections, any other punishment has objections ; hence no punishment is suitable to protect the safety of the citizens.

9. Fallacies Incidental to Induction. The steps followed in inductive inference are : Firstly the collecting of facts, secondly the interpretation of them by suitable hypotheses, and thirdly an inference or generalisation. Each is liable to a peculiar wrong process.

Fallacy of Simple Inspection. In collecting facts there may be Non-observation and Mal-observation. Some logicians take an imperfect observation to be rather a mistake than a fallacy. I see no objection to calling it a fallacy in the sense that wrong judgments follow from faulty sense-perceptions, and develop from them. Examples :

1. Until modern science developed, it was generally supposed that the heavenly bodies rotated around the earth, causing the succession of day and night.—The fact, however, was taken for granted without any test. The deception came from taking appearance for a fact, or non-observation.

2. The spontaneous generation of little organisms from inorganic matter was an admitted fact among the Schoolmen.—They made no attempt at verification. Another instance of non-observation.

3. Fossils of plants and animals, because of their appearance, were looked upon by ancient writers as playthings of nature, *lusus naturae.*—A mere fancy taken for a fact.

4. Statistics are often gathered in a hasty manner. Testimonials of patients, cured by a certain medicine used by them, are taken as facts to obtain the patent certificate. —Are these facts reliable ? They will be so if the cases are similar, if no other medicine has influenced the recovery, and above all if the negative cases are taken into account. Without these precautions it is a case of Mal-observation.

5. A person habitually possessed by fear, or overnervous, takes a sudden noise or a mere shadow for a ghost or an enemy.—He sees something, but not that. Again, Mal-observation.

6. Prejudices, feelings of pride and anger, a great desire for a discovery, too often make one see in another person, in an enterprise, in the microscope, that thing which is most in his own mind.

False Cause, or *Post hoc, ergo propter hoc.* Untrained minds easily suppose a cause in what is a mere antecedent, or a condition. In like manner they establish a fixed relation where they see only a co-existence. And generally speaking they put down as a true explanation what is only a probable one. Examples :

1. Changes of weather follow or coincide with the successive occurrence of full moon and new moon. Therefore the moon is the cause of those changes.—No one can point out a fixed succession or concomitant variation. Hence no such causation is proved.

2. An epidemic happens to rage in the locality at the time of an eclipse of the sun, or at the appearance of a comet. Therefore, the vulgar mind says, that must be the cause of the epidemic.—All superstitions are likewise instances of a false cause.

False generalisation, as the name indicates, means to conclude that a relation holds universally, because it has been observed in some cases. Examples :

1. A few students have been found to be mischievous, and they are connected with a certain institution ; therefore, there is no discipline in that institution.—Observe that simple enumeration cannot be the ground of universality, it only suggests a hypothesis. The conclusion extends as far as the number of cases.

2. We often hear of a fraud or forgery by someone who could not have committed such a crime if he had never learned to read or write. Hence a condemnation of all teaching.—Teaching is here declared generally bad, because of a probable connection of particular teaching with particular persons.

3. We see cases of misery and crime due to the use of ardent spirits. Therefore, there should be a general law of prohibition.—But this misery and crime is connected with an *abuse* of ardent spirits, not with the *use*. Hence there is no ground for the prohibition, much less for a general prohibition of the use of ardent spirits.

4. Switzerland is a republic and a peaceful country, the United States is also a republic and politically well settled. Therefore, no revolution will develop under a republican government.—Illicit generalisation.

5. An excise officer finds that three bottles of wine, picked at random cut of a quantity imported, agree in strength ;

therefore he concludes that all the bottles imported are of the same strength.—He may conclude that way probably, but not certainly.

Proving too much. An old saying indicates this fallacy: "He who proves too much, proves nothing." Why is this? The conclusion that proves too much is one that extends beyond the range of true facts, i.e. false generalisation not guaranteed by the premises. Hence the premises do not prove the conclusion, for a falsehood cannot be proved. Example:

Any system of correction other than appealing to reason should be avoided, because it is below man's dignity. But no corporal punishment appeals to reason, but to the senses; therefore, corporal punishment should be universally avoided.—The argument proves too much, by excluding all cases under any condition from corporal punishment. There are evidently many cases deserving punishment where appeal to reason is impossible, as in the wrongdoing of children, of insane people, of great criminals. The conclusion cannot be proved by the premises, for the premises are not unconditionally true. Corporal punishment is indeed the last thing to be resorted to, and never without measure or reason. Under these conditions it is not below man's dignity, but rather above, for it brings man back to his dignity.

10. Final Remark. Observe that the above list of fallacies is not exhaustive, nor are the examples given so fixed under one head that they cannot be classed under another, if looked at from a different point of view. In reality all fallacies correspond to one or other of the three elements of inference. The fault of an argument, if any, is either in the premises, or in the conclusion, or in the connection between them. One may simplify the whole matter further by stating that every deductive fallacy is simply a wrong proof. Thus Monck says: "It is no easy task to treat of fallacies within a moderate compass, especially as writers on Logic are not agreed as to what is to be called a *Fallacy*. The distinction which I would draw is a very simple one. I would define a fallacy an *invalid argument*, or an argument in which the conclusion does not follow from the premises." [1]

The analysis of fallacies, however, has a great advantage. Looking into the ways in which people think wrongly, and laying side by side the right principle with its wrong application, the reader becomes cautious and understands better how he ought to think, which is the main purpose of Logic.[2] Let us conclude as follows :

The best way of avoiding fallacies is to have a sincere and impartial love of truth, no matter what one's personal interest or prejudices may be. The best way of escaping from the fallacies of others is the same sincere and impartial love of truth quickened by cautious vigilance. "Prove all things and hold fast to that which is good." [3]

One of the attributes of God is that He neither deceives nor can be deceived. Man's mind, created in the image and likeness of God, is meant to reflect both qualities. Therefore in all our processes of reasoning let us aim at the truth, the whole truth and nothing but the truth, and be on our guard against perpetrating fallacies ourselves, and so we shall not deceive others. Let all that we hear from others be submitted to examination with the same end in view, and so we shall detect their fallacies and shall not be deceived by others.

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[1] W. H. S. Monck, *An Introduction to Logic*, p. 74.

[2] For illustrations of practical logic, the reader may consult Balmes, *The Art of Thinking Well*, last chapter, on practical understanding. He develops the idea that sound logic should comprehend the whole man ; for truth is in relation with all the faculties of man.

[3] St. Paul, 1 *Thess.*, V. 21.

QUESTIONS AND EXERCISES

Many of these questions are taken from Examination Papers.

CHAPTER 1

1. Can you account for the natural origin of Logic as we do in other sciences? State the particular subject matter of Logic.

2. Outline the history of Logic, and point out the thinkers who in ancient and modern times have contributed to expand the logical doctrine.

3. Define Logic, and explain the terms used so as to bring out the nature of the subject. Criticise these definitions :

   (*a*) Logic is the science of thought.
   (*b*) Logic is the science of reasoning.
   (*c*) Logic is the science of the operations of the understanding which are subservient to the estimation of evidence.

4. What is meant (*a*) by form, and (*b*) by matter in Logic? Illustrate how the form of different propositions may be identical whilst the matter may vary.

5. Which do you take to be the most appropriate Division of Logic?

6. Explain the scope of Logic and its general character. Can it be maintained that the field of Logic is co-extensive with the field of knowledge? Discuss.

7. " Logic is a normative science." Explain this and illustrate the difference between normative and positive sciences.

8. Distinguish consistency of thought from Logical Truth, and show with examples that either may exist without the other.

9. " The aim of Logic is logical truth." Discuss this, and consider the various states of mind in regard to truth.

10. Describe the activities of the mind which are at work in the attainment of truth. Is it the province of Logic to dictate rules for all the cognitive faculties?

11. Compare the characteristics of Science with those of Art, and accordingly decide whether Logic is rather a science than an art or both.

12. Logic has sometimes been called the Science of sciences. In what sense are all the other sciences part of the subject matter of Logic ?

13. In what way or ways is Logic related (*a*) to Language in general, and (*b*) to Grammar in particular ? Explain.

14. " Logic deals with language as well as with thought." Discuss.

15. A mathematical formula is a form of thought. On this supposition do you maintain that Mathematics has a logic of its own and distinct from ordinary Logic ? Explain.

16. Explain the particular relation that exists between Logic and Psychology.

17. " Logic does not teach us how to think." Discuss this statement, and explain the utility of Logic.

CHAPTER II

1. Explain the Simple Concept and its degrees of perfection. Which concepts are usually indistinct in the mind ? Give examples.

2. Distinguish between a Word, a Name and a Term. How are the three related to the concept ?

3. " Logic looks at the Parts of the Speech in a manner which differs from that of Grammar." Discuss.

4. State the Parts of the Speech which are of primary, and those which are of secondary importance in Logic.

5. Consider the difference (if any) between a Concept, a Name and a Term.

6. What are the Logical Characters that may be found in any name ? Explain the precise meaning of Singular and General names.

7. Compare the Collective and the Distributive use of a name. Give three examples of names used collectively and distributively.

8. Are Substantial names singular, general or collective ? Give your reasons.

9. State the logical characters of the following names :

> Chemistry, garden, dozen, number 6, the sky, language, building, prime-minister, city, collection, road, book, silver, this pencil, the north pole, His Majesty, alphabet, society.

10. Explain the distinction between Abstract and Concrete names. Apply this distinction to the following :

> Illness, honesty, falsity, description, employment, socialism, progress, complexity, convenience, loyalty ability, strength, efficiency, pleasure, nationality, friendship, distinction, kindness.

11. "Names grammatically considered, or taken by themselves, may be abstract or concrete, but in a context all names are concrete." Discuss.

12. What are Positive and Negative names ? Are there names which are purely negative ? Examine the following .

> Darkness, nonsense, unloosed, unchangeable, unimpaired, unequal, obscure, dishonesty, hatred, nothing invisible, deafness, blackness.

13. Has every name its Contrary and its Contradictory ? Give, if possible, a contrary and the contradictory of each of the following :

> Empty, equal, friend, simple, unpleasant, unhealthy, present, insanity, convalescent, changeable, intelligible.

14. How would you decide whether two names are contrary or contradictory ? Examine in accord with the " universe of discourse " the opposition between: true and false, moral and immoral, healthy and unhealthy, black and white, careless and careful, agreeable and disagreeable, present and absent, young and old, valid and invalid, empty and full, certain and uncertain, pleasant and unpleasant, honest and dishonest.

15. Define Incompatibility of names and its various degrees. State the kind of incompatibility (if any) between the first and each of the following :

> Useful, harmful, harmless, pleasant, useless, inconvenient, suitable.

16. Is the subject-term of the following sentences singular, general or collective ?

(1) The man I refer to is honest.
(2) Sixteen annas make a rupee.
(3) These books can fill a box.
(4) These books belong to my library.
(5) The crew saved all the passengers.
(6) The students of this class are all Indians.
(7) Air is a gas.
(8) Air is the atmosphere of the earth.
(9) Air is a word of three letters.
(10) The president of the republic is elected.

17. State whether the subject-term in the following sentences is used collectively or distributively :

(1) Two wrongs will not make a right.
(2) All these claims upon my time overpower me.
(3) The soldiers surrounded the building.
(4) The fine arts are fine.

(5) The fine arts are a source of entertainment.
(6) All the angles of a triangle are less than two right angles.
(7) The mob was dispersed.
(8) The court had passed sentence.

18. Compare the logical characters of the names in each of the following groups :

(1) Men, human nature, mankind.
(2) Foolish, fool, folly.
(3) Colonel, regiment, soldier.
(4) Mountain, the highest mountain, Mount Everest.
(5) Good, better, best.
(6) A soldier's life, military life.
(7) Art, artistic, artificial.
(8) Sea monsters, marine monsters.

19. Distinguish between the Connotation and Denotation of names. State the meaning in connotation and in denotation of each of the following :

A five-rupee note, my parents, teacher, colour, judge, the present Governor of Bombay, motor car, university student, mistake.

20. Discuss the following statements :

(*a*) Every name has both connotation and denotation.
(*b*) Proper names are meaningless marks.
(*c*) It is impossible to have a name without denotation.

21. Explain the peculiarity of Attributive names, their connotation and denotation.

22. What is the precise connotation and denotation of (*a*) abstract names, and (*b*) negative names ? State the connotation and denotation of the following :

Blind, simplicity, blackness, unemployed, colourless, criminality, indifference, bad, worse, worst.

23. How far is it true that connotation and denotation are inversely related ? Exemplify your answer with the following names :

(*a*) Engine, steam-engine, mechanism, heavy mechanism, composite body, body, thing, this steam-engine.
(*b*) Map, figure, drawing, map of India, the largest map of India.

24. State the various meanings or kinds of connotation and their practical use.

25. What is first in a name, connotation or denotation ? Who determines the connotation of names ? And is the connotation of names fixed for ever ?

26. Compare the connotation and the denotation of the names in each of the following groups :

(1) Poison, poisoned, poisonous.
(2) Employed, non-employed, unemployed.
(3) Captain, captaincy, Captain Smith.

27. ' Any term whatever in a context must have both connotation and denotation." Discuss.

28. What is meant by Ambiguity of names, and how does it come about ? State the precise meaning of the word " simple " in the following expressions :

Simple instrument, simple question, simple character, simple style, simple room, simple manners, simple mind, simple product, simple business.

29. Explain the meaning of " universe of discourse " or supposition", as applied to names. Illustrate your answer with the following examples :

(1) The High Court will give a final decision.
(2) The High Court is in need of repairs.
(3) Coins are metallic.
(4) Coins is a noun.
(5) Coins are a kind of currency.

30. Give examples of Univocal, Equivocal and Analogous names. State fully the logical characters of the following names according to their various meanings :

Board, sound, museum, lake, atmosphere.

31. Show that the following statements are all true or all false according to the meaning given to a single word :

(1) All men are equal.
(2) No two men are equal.
(3) Compulsory education is advisable.
(4) Compulsory education is not advisable.
(5) It is impossible for all people to avoid crime.
(6) It is possible for all people to avoid crime.

32. " Names in a given context can have one meaning only." What is the use, therefore, of so many divisions of names ? Explain.

CHAPTER III

1. Give an account of Mental Abstractions, and illustrate the Universality of our concepts.

2. What do you understand by Predicables ? Explain the principle underlying the five-fold scheme of predicables.

3. Give a definition and also an illustration of each of the five predicables.

4. Explain and illustrate the distinction (*a*) between essence and accident, and (*b*) between difference and property.

5. Why is it (*a*) difficult and (*b*) important to distinguish in practice between a property and an accident ?

6. State the kind of predicables that may be assigned : (*a*) to a singular name, (*b*) to a general name (species), (*c*) to a general name (genus), (*d*) to a proper name. Justify your answer.

7. Compare : clerk with employee, assembly with parliament, railway with road, money with wealth, capital with city, captain with officer, game with exercise ; and say in each case which is a genus, which a species, and what is the difference.

8. Show that genus, species and difference are related to one another. Is the genus contained in the species, or the species in the genus ? Give examples.

9. Compare the fine predicables in connotation and in denotation. Point out the predicables which always agree in denotation. Illustrate your answer.

10. Explain with examples : lowest species, proximate genus, remote genus, highest genus ; and show that one and the same name can stand for logical genus and species.

11. Explain and exemplify : specific property and generic property, inseparable accident and separable accident.

12. State the five predicables, or as many as possible, in regard to each one of the following terms :

The monsoon, water, this watch, salt, Governor of Bombay, Goa.

13. What is a Predicamental line ? Attempt the predicamental line of each of the following names :

Circumference, apple, health, the earth, steam-engine.

14. Under which of the predicables would you place the predicates of the following sentences :

(1) A little learning is dangerous.
(2) An essay is a written composition.
(3) Sunday is the first day of the week.
(4) Agriculture produces wealth.
(5) Copper is a conductor of electricity.
(6) Poets are irritable.
(7) Tree is a noun.
(8) My friend is ill.
(9) Boys are capable of education.
(10) This man is six feet high.
(11) Coffee is a stimulant.
(12) A libel is a malicious statement.
(13) Veracity is truthfulness.
(14) The captain of the ship is Robert Smith.
(15) Paris is the capital city of France.
(16) Mount Everest is the highest peak on earth.

15. Illustrate with the following sentences the distinction between "generic" and "specific" property, and between "proximate" and "remote" genus :

(1) Ducks are web-footed.
(2) Honesty is a personal quality.
(3) A person is a living being.
(4) A five-rupee note is a kind of money.
(5) The moon is a material body.
(6) Examinations are a mental exercise.
(7) Books are artificial productions.
(8) All machines are liable to decay.

16. Give an account of the Aristotelian Categories and exemplify them.

17. State the difference between a Category, a Predicable and a Predicate.

18. Consider the use and value (*a*) of predicables, and (*b*) of categories for the purpose of Logic.

19. Explain the purpose of Definition. Are definitions a necessary means of knowledge ?

20. What are the essential characteristics of a good definition ? Consider how definition is related to predicables.

21. Enumerate the various kinds of definition. Give examples.

22. Criticise the definitions in each of the following sets :

(1) Food is what is sold in the market. Food is such things as rice, bread, beans and the like. Food is a nourishing substance.
(2) A libel is a malicious statement. A libel is an injurious statement. A libel is an injurious publication.
(3) Art is a kind of work like painting, music, carpentry. Art is a kind of knowledge of how things are made. Art is production of work in accord with a code of rules.
(4) A proverb is a common saying. A proverb is a sentence containing a well-known truth. A proverb is a short sentence which forcibly expresses some practical truth.

23. State and illustrate the logical rules for a sound definition.

24. Examine the following definitions in accordance with logical rules :

(1) A pharmacy is a drug store.
(2) Blue is the colour of the clear sky.
(3) A friend is one who is friendly to another.
(4) A book is a source of learning.
(5) A clock is an instrument that measures time.
(6) Pleasure is the opposite of pain.

(7) Sloth is the key to poverty.
(8) Gratitude is the fairest blossom that springs from the heart.
(9) A soldier is a brave man ready to die.
(10) Peace is the absence of war.
(11) Capital is that part of the wealth of a country which is employed in production.
(12) Zero is a number which is not the successor of any natural number.

25. Attempt the analytic definition, or else a description of each of the following names :

Picture, advertisement, monument, lake, arm-chair, tram-car, telephone, dictionary.

26. " Some names are capable of many definitions, while other names are indefinable." Discuss this, and point out the limits of definition.

27. Explain Logical Division and its relation to definition.

28. State and illustrate the rules of a sound division.

29. Describe: Enumeration, Physical Partition. Moral Division, Conceptual Division; and show how they differ from logical division.

30. Examine by the logical rules the following attempts at division :

(1) People into civilised, poor, industrious and employed.
(2) Trains into local and electric.
(3) Chair into seat, back and arms.
(4) Fine arts into painting, music, sculpture, architecture, literature.
(5) Person into flesh, bones, feelings and thoughts.
(6) Men into European, Asiatic, Indian and Portuguese.

31. Illustrate Division by Dichotomy, and show by reference to the rules its practical utility.

32. Attempt a complete logical division of each of the following names :

Coins current in India, temples, schools, buildings, motor cars, chairs, pictures.

33. " Definitions and likewise divisions are provisional and subject to revision." Explain.

CHAPTER IV

1. Explain from a logical point of view how the Second Operation of the mind is an advance on the simple concept.

2. Give examples of perfect and imperfect judgments. Why is the judgment said to be the Unit of knowledge ?

3. Compare the judgment with the proposition, and distinguish between a proposition and a sentence.

4. Explain the logical proposition, and mark the conditions whereby the subject may be distinguished from the predicate. Which of the two is the more important ?

5. State the function of the logical copula. Is the copula any reality distinct from the terms of a proposition ?

6. Compare the logical copula with the verb in a proposition, and distinguish the time " of " predication from the time " in " predication.

7. What is meant by putting a statement in strict logical form ? Illustrate the difference between the grammatical and the logical subject and predicate.

8. What is meant by truth and falsehood in a proposition ? Examine these statements :

(*a*) Every proposition is either true or false.
(*b*) A proposition once true always true, once false always false.

9. Show that the following: (*a*) the Governor is in Bombay, (*b*) the Governor is not in Bombay, are both true or not according to the time " in " predication.

10. Explain the Quantity and Quality of the logical proposition, and how these two elements give rise to the traditional forms of propositions.

11. Re-state the following sentences in their strict logical form, and add the symbols **A, E, I, O** :

(1) Every mistake is not a proof of ignorance.
(2) It is more blessed to give than to receive.
(3) All men are not liars.
(4) Few men are acquainted with themselves.
(5) Not every advice is a safe one.
(6) A few are not disheartened by failure.
(7) All is not true that seems so.
(8) Nothing is harmless that is mistaken for virtue.
(9) It is 75 miles to Poona.
(10) Hardly any nation is free from economic distress.
(11) Nothing conduces more to breadth of intellect than intercourse with various minds.
(12) The best thing you can do for the people is to make them jolly.
(13) Now is the time.
(14) It is a fine day.

12. Explain the precise meaning of the words " all " and " some " as logical signs of the quantity of propositions. How do you quantify singular propositions ?

13. State clearly the quantity and quality of the following propositions :

(1) A few drops of rain are not of much consequence.
(2) Few men succeed in life.
(3) Man is a word of three letters.
(4) Great are the glories that surround a throne.
(5) Many rules of grammar overload the memory.
(6) Two blacks don't make a white.
(7) When the king commands anything it is done.
(8) Ten pounds is a lot of money.
(9) It does not always rain.
(10) These claims upon my time overpower me.
(11) There are students and students.
(12) He envies others' virtue who has none himself.
(13) No friends are like old friends.
(14) There are foolish politicians.

14. How would you decide the quantity : (*a*) of indesignate propositions, (*b*) of numerical subjects, (*c*) of propositions affected by secondary quantifications ?

State the quantity of the following :

(1) Half of the passengers were saved.
(2) There are three things to be considered.
(3) They never fail who die in a great cause.
(4) There is at least one student who is not present.
(5) Light minds are pleased with trifles.
(6) He jests at scars that never felt a wound.
(7) Things are subject to change.
(8) No man ever fails to remain poor who is both ignorant and lazy.
(9) Children are capable of education.
(10) People are sometimes forgetful.

15. What is meant by distribution of terms in a categorical proposition ? Show that the distribution of the predicate depends upon the quality of the proposition. Examine the distribution of terms in the following :

(1) He always succeeded.
(2) Vice never brings happiness.
(3) All scientific books are not difficult.
(4) Home is always in the background of our thoughts.

16. Discuss the view that all sentences can be forced into logical form. What difficulties do you meet with in trying to translate: (*a*) proverbial phrases, (*b*) questions, (*c*) sentences expressing wish or command, into logical form ?

17. Transform the following sentences as far as possible into their logical form :

(1) What I have written, I have written.

(2) With time I have no quarrel.
(3) Business is business.
(4) Times are changed.
(5) Lay aside your complaints.
(6) How changed from what he was !
(7) Consistency, thou art a jewel !
(8) Whom the cap fits, let him wear it.
(9) A stitch in time saves nine.
(10) What region of the earth is not full of our labour ?
(11) I wish I had not done it.
(12) Do unto others as you would have others do unto you.
(13) Press not a falling man too far.
(14) Put your shoulder to the wheel.
(15) How poor are they that have no patience !
(16) Let nothing but good be said of the dead.
(17) Meddle not in family affairs.
(18) Can the leopard change his spots ?
(19) We know things chiefly by contrast with their opposites.

18. State briefly the main questions involved in the Import of Categorical propositions.

19. Explain and illustrate the various interpretations of propositions as to the connotation and denotation of subject and predicate.

20. Apply to the following propositions the interpretation that you consider most appropriate :

(1) All politicians are not statesmen.
(2) No man is infallible.
(3) No news is good news.
(4) Honesty is compatible with ignorance.

21. Discuss the logical value of the Class view and of the Predicative view.

22. Examine the doctrine of the Quantification of the predicate. Can the logical copula be substituted by a sign of equality ?

23. Examine whether a proposition implies a relation (*a*) between words, or (*b*) between concepts, or (*c*) between things.

24. In what sense is every proposition a statement of reality ?

25. Discuss the question whether categorial propositions imply the existence of either subject or predicate.

26. Which theory of the import of propositions is at the base of the four-fold scheme of propositions ? Explain.

CHAPTER V

1. What is meant by a Complex categorical proposition ? Show by examples how adjectives and phrases modify the connotation and denotation of terms.

2. What are Modal propositions ? Explain the kinds of modality and their effect upon the quality and quantity of a proposition.

3. Express the following propositions in logical form (*a*) without modality, and (*b*) with modality :

(1) Old things may not be the best.
(2) Steamers may perish in a stormy sea.
(3) Propositions must be true or false.
(4) Politicians need not be graduates.
(5) No one can be learned who is not both studious and ambitious.
(6) You cannot teach an old dog new tricks.

4. Explain the conditions of logical modality, and how it differs from a mere subjective belief.

5. Criticize Kant's division of propositions into Assertive, Apodictic and Problematic.

6. Illustrate the various kinds of Exponible propositions. How is the meaning of such propositions logically analysed ?

7. Express each of the following sentences in their logical form :

(1) Only experts can judge of scientific matters.
(2) Only children are not admitted.
(3) Only conservatives voted for the bill.
(4) Only non-combatants are not allowed within the firing line.
(5) Nothing but coolness could have saved him.
(6) No admittance except on business.
(7) Few metals only are not solid.
(8) Some students only are not prepared for examination.
(9) Dead languages are not the only ones worth studying.
(10) There is one thing only which gathers people into seditious communities, and that is oppression.
(11) I think it is better to restrain children through a sense of shame and by liberal treatment than through fear.

8. What is a Compound categorical proposition ? And how is it logically examined ?

9. Analyse logically the following compound propositions :

(1) Virtue consists neither in excess nor defect of actions, but in a certain mean degree.
(2) Threats may sadden, but they never convince.
(3) Many are called, but few are chosen.

(4) It is not the quantity of the meat but the cheerfulness of guests which makes the feast.
(5) " A gentleman makes himself agreeable to all, never attracts attention, is never loud, never hurts others' feelings, is usually generous and is always just." (Newman.)
(6) Alms-giving never made a man poor, nor robbery rich, nor prosperity wise.

10. Explain the distinction between Analytic and Synthetic, Verbal and Real, Explicative and Ampliative, *A priori* and *A posteriori* propositions.

11. Discuss, with examples, the logical foundation of Kant's prepositions " Synthetic *a priori.*"

12. Consider the relation of analytic and synthetic propositions to the doctrine of the predicables.

13. Decide whether the following propositions are analytic or synthetic, and give a reason :

(1) It takes two to make a quarrel.
(2) Liquids have no shape of their own.
(3) Not all brilliant speeches are a proof of wisdom.
(4) Man is a substantive noun.
(5) Four is greater than three.
(6) Oppressed people are discontented.
(7) Calcutta is a great city.
(8) Honesty is the best policy.
(9) Few men reach the age of 90.

14. Explain the nature of a Hypothetical proposition. When is a hypothetical proposition logically true ?

15. How would you decide the quantity and quality of a hypothetical proposition ? State the following in their hypothetical form :

(1) Till the nature of the offence is known, the law cannot be set in motion.
(2) A ready way to lose your friend is to lend him money.
(3) We must not deduce an argument against the use of a thing from an occasional abuse of it.
(4) No man can be happy, unless he is temperate, and not always then.
(5) Every man of military age, unless he be exempted as physically unfit, or as indispensable to some work of national importance, is sent into training.

16. Compare the categorical with the hypothetical proposition, and discuss the relation between the two.

17. Can a categorical proposition be satisfactorily translated into hypothetical ? Point out the distinction between Empirical and Generic categorical propositions.

18. Are the following propositions empirical or generic? Express them hypothetically:

(1) All plants are green.
(2) False friends are worse than open enemies.
(3) None of the islands of Europe is very large.
(4) Ill-founded enmities are ever the most obstinate.

19. Change the following hypothetical propositions into the three-term form, and into categorical:

(1) If the sea is rough, fishermen are in danger.
(2) If the monsoon is bad, the harvest will be poor.
(3) If this work requires three hours a day, I cannot do it.
(4) If there is a will, there is a way.

20. Discuss the nature of a true disjunctive preposition so as to bring out its precise and necessary import.

21. What different views have been taken as to the import of a disjunctive proposition? Explain.

22. State the relation between hypothetical and disjunctive propositions. Express hypothetically:

(1) Either A is not-B, or C is D.
(2) A is either B or not-C.
(3) Either thou art most ignorant by age, or thou wert born a fool.
(4) Either a man is virtuous or he is unhappy.
(5) Either he has forgotten, or he is deliberately lying.
(6) A man is either studious, or he is not learned.

23. Change the following into disjunctive form:

(1) If anything is not-P, then it is Q.
(2) If A is not-B, then C is not-D.
(3) If a university gives stimulative teaching, it does everything.
(4) If you grasp at too much, you will catch nothing.
(5) If any work not, neither should he eat.
(6) If you command wisely, you will be obeyed cheerfully.

24. What is the import of a conjunctive proposition expressing incompatibility? State the logical import of the following:

(1) Not both P and not-Q.
(2) You cannot be both a royalist and a republican.

25. Given: If not-P, then Q; if not-Q, then not-P; either not-P or Q; if not-Q, then P; not both P and not-Q; all P is Q; find out which of these propositions are equivalent to "if P, then Q", and which to "either P or Q."

26. Consider the following propositions, and show that they are equivalent to one another: (*a*) every S is P; (*b*) nothing is both S and not-P; (*c*) if anything is S, it is P; (*d*) either not-S

CHAPTER VI

1. Explain the general character of the Laws of Thought. If these laws are inviolable, what is the source of fallacy ?

2. State the traditional Laws of Thought. Are these laws incapable of proof ?

3. What is the difference between the Law of Contradiction and the Law of Excluded Middle ? Consider the conditions necessary to their validity.

4. Is the Law of Contradiction applicable to the propositions : (*a*) Some boys are mischievous ; (*b*) Some boys are not mischievous ?

5. Point out the Law of Thought which is implied in the following expressions :

(*a*) Every proposition is either true or false.
(*b*) From nothing nothing can be made.

6. " The number thirteen is not unlucky, therefore the number thirteen is lucky." Is this a case of Excluded Middle ?

7. Explain the significance of the Law of Excluded Middle. " That Mars is inhabited is neither true nor false." Does it mean that the Law of Excluded Middle fails ?

8. Criticise the following cases of Excluded Middle :

(*a*) Either I shall fail or pass in the examination.
(*b*) Every thing is either black or of another colour.
(*c*) Any proposition is either known to be true or known to be false.

9. Explain the precise meaning of the Law of Identity. Mention one or two instances of its application in the course of Logic.

10. Consider to what extent the Law of Causation differs from the Law of Sufficient Reason ; and point out the sphere of their application.

11. What is the function of the Laws of Thought ? Do they afford more than a negative test for the validity of our reasonings ?

12. Consider the relation, if any, that exists among the traditional Laws of Thought. Are they interdependent ?

13. Explain the notions : Principle, Ground, Reason, and their use in Positive and in Negative proofs.

14. What are the essential conditions for a logical Inference ? Is it possible to obtain a logical conclusion from a false statement ?

15. Illustrate the various kinds of Opposition. Which is the most important ? Has every proposition its contrary and its contradictory ?

16. Prove that two Contradictory propositions can neither be true nor false together.

17. State and prove the rule of inference between two Subaltern propositions.

18. Prove that Sub-contrary propositions (*a*) cannot be false together, (*b*) may be true together.

19. State and prove the law of inference between Contrary propositions. Can the following propositions be (*a*) both true, and (*b*) both false ?

(1) No man is a liar. (2) This man is a liar.

20. Give all the logical opposites of each of the following :

(*a*) No tale-bearer is to be believed.
(*b*) Every mark of weakness is not a disgrace.

21. Examine the opposition between the following :

(1) There are logical authors who are scientific.
(2) It is untrue that all logical authors are scientific.
(3) Those who are logical authors are all scientific.
(4) Not one of the logical authors is scientific.

22. Given as true : "Nothing is difficult to a willing mind", state the propositions that follow as true, and those that follow as false from opposition.

23. Given as false : " Only clever boys are fit for education," lay down the propositions which may be obtained as true, or as false by opposition.

24. State, giving examples, what can be asserted as to the truth of a proposition from (*a*) the falsity of its contrary, (*b*) the truth of its subalternate, (*c*) the falsity of its contradictory, (*d*) the truth of its subcontrary.

25. What is the precise difference between contrary and contradictory propositions ?

State the contradictory and (where possible) the contrary of each of the following :

(1) Not one of the enemy escaped.
(2) Sometimes he abates these claims.
(3) Anyone but a fool would see this.
(4) Two-thirds of the army are abroad.
(5) A few of our employees are not honest.
(6) The *Examiner* is published only on Sundays.
(7) Many hands make light work.

26. If you cannot truly say :

(1) There's luck in odd numbers,
(2) Most people are free from vanity,
(3) Ignorance is always bliss,
(4) Only graduates succeed in life—what can you say in truth ?

27. Give the contrary of each of the following :

(1) Only the ignorant affect to despise knowledge.
(2) The uneducated alone are not fit to vote.
(3) It is impossible that all do all things.
(4) No occupation fails to be elevating if pursued with sincerity.
(5) He is always late.
(6) No man is perfect who desires not greater perfection.

28. Is every proposition, categorical, hypothetical or disjunctive, capable of all kinds of opposition ? Discuss.

29. A proposition may be opposed by a contrary or by a contradictory, but the latter way is easy and safe, while the former is difficult and unsafe. Explain.

30. What is Eduction ? State and justify the rules of Obversion and Conversion. Why is it that an **O** proposition cannot be converted ?

31. Give the converse and the obverse of each of the following :

(1) Whatever is necessary exists.
(2) No man is perfect who desires not greater perfection.
(3) There is no man in the world without some trouble.
(4) Times of calamity have always been productive of the greatest minds.

32. Describe briefly Contraposition and Inversion. How would you prove that both are a legitimate process ?

33. Show how a statement may be expressed formally as **A, E, I** or **O** proposition.

34. Can you assert the truth of a proposition from the truth of its simple converse or its obverse ? Give a reason.

35. If you say : "No unjust acts are profitable," make a list of the propositions to the truth of which you commit yourself.

36. Given : " All intemperate people are unhealthy," what can be truly said (*a*) of unhealthy, (*b*) of healthy, and (*c*) of temperate people.

37. Transform the following propositions, so as to have S as subject and P as predicate :

(1) No not-P is S.
(2) All P is not-S.
(3) Some not-P is not not-S.
(4) All not-P is not-S.

38. In the proposition : " All things mortal are material " reverse the terms from positive into negative.

39. Examine the logical relation between (*a*) and (*b*) in the following pairs of propositions :

(1) (*a*) Only experts are judges in scientific matters.
(*b*) All judges in scientific matters are experts.

(2) (*a*) All not-S is P.
    (*b*) All not-P is S.
(3) (*a*) Some stories are true to fact.
    (*b*) Some stories are other than true to fact.

40. Work out, where possible, (*a*) the contrapositive, and (*b*) the inverse of the following propositions:

(1) Whatever is certain is infallible.
(2) No theory is unreservedly accepted.
(3) Every truth is not to be told.
(4) Some able people are unfortunate.

41. Point out the relation between the proposition: "No knowledge is useless" and each of the following:

(1) All knowledge is useful.
(2) Something useful is knowledge.
(3) Some knowledge is useful.
(4) What is not useful is not knowledge.
(5) What is not knowledge may be useless.
(6) Not all things useful are other than knowledge.
(7) All knowledge is not useless.

42. If you admit that "Some teachers are not graduates," you must agree to hold that "Some graduates are not teachers." Give a reason for your answer.

43. Explain the words Consistent, Inconsistent, Inferrible as applied to propositions. Give two examples of propositions quite consistent and not inferrible.

44. State whether (*a*) and (*b*) in the following pairs of propositions are consistent; if consistent, state whether either is inferrible from the other:

(1) (*a*) No virtue is ultimately injurious.
    (*b*) Nothing ultimately injurious is a virtue.
(2) (*a*) All philosophers are clever thinkers.
    (*b*) Some clever thinkers are not philosophers.
(3) (*a*) This climate always restored my health.
    (*b*) What failed to restore my health was never this climate.
(4) (*a*) Only foreigners are not eligible to that post.
    (*b*) All eligible to that post are foreigners.
(5) (*a*) Not only fools learn by experience.
    (*b*) No one who learns by experience is a fool.

45. Assuming the proposition: "All skilful persons are ambitious", decide which of the following are true, and which false:

(1) No skilful persons fail to be ambitious.
(2) Some who are not ambitious are skilful.
(3) Some who are not skilful are unambitious.

(4) All unskilful persons are ambitious.
(5) All unambitious persons are unskilful.
(6) All skilful persons are other than ambitious.

46. Are eductions applicable to hypothetical propositions? Give, if possible, the converse, the obverse, and the inverse of each of the following:

(1) If any man is dishonest, then never is he trusted.
(2) If any production is artificial, then always it is imperfect.
(3) If a speech is brilliant, then sometimes it is convincing.
(4) If a story is entertaining, then sometimes it is not true.

47. Explain the distinction between Formal and Material immediate inference.

48. Examine the nature and validity of the inferences:

(1) Whoever works hard deserves to pass; therefore whoever deserves to pass works hard.
(2) A is to the right of B; therefore B is to the left of A.
(3) No idle persons can succeed in life; therefore all industrious persons are successful in life
(4) A slave is a man; therefore he who murders a slave murders a man.
(5) If not-Q, then not-P; therefore if P, then Q.
(6) Not both P and Q; therefore if not-Q, then P.
(7) Either not-P or Q; therefore all P is Q.
(8) Uneasy lies the head that wears a crown; therefore easy lies the head that wears no crown.
(9) The pen is mightier than the sword; therefore the sword avails less than the pen.
(10) All civilised persons are polite in manners; therefore impolite persons are uncivilized.
(11) No one is admitted without payment; therefore all who are admitted are persons who paid.
(12) Human productions are all imperfect; there must be therefore some things other than human productions which are perfect.
(13) Lazy persons are often out of work; therefore a man out of work is lazy.
(14) Whatever is false leads to evil consequences; therefore whatever leads to good consequences is true.
(15) All men are fallible; therefore the universal assent of mankind is not a test of truth.
(16) Every man has a right to the product of his own labours; therefore no man has a right to what is not the product of his own labours.
(17) Prudence is a virtue; therefore a prudent man is a virtuous man.

(18) No man can practise as a lawyer unless he passes the Law examination ; therefore all who pass the Law examination are practising lawyers.

49. What claims has Immediate inference to be called inference ? Discuss.

CHAPTER VII

1. Explain and illustrate the general character of Mediate inference. What are the main kinds of mediate inference ?

2. (*a*) Inference is the process of unfolding the implications contained in the antecedent propositions.

(*b*) Inference is the process by which we pass from the known to the unknown. Criticise the above statements.

3. State the precise meaning of the terms : Reasoning, Inference, Proof, and their relation to one another.

4. (*a*) Inference is neither true nor false, but valid or invalid. (*b*) Valid inference is compatible with a false conclusion. Explain.

5. State what you know about the truth of the premises in a valid inference (*a*) from the truth of the conclusion, and (*b*) from the falsity of the conclusion.

6. Define the categorical syllogism, and show that the Middle term holds that relation to its extremes which the name implies.

7. Prove the validity of syllogistic reasoning, and show that the syllogism is a formal inference.

8. State the respective function of the major and minor premise. If the conclusion of the syllogism is implied in the premises, does it give any new knowledge ?

9. Apply the axiom " Dictum de omni et nullo " to the following premises and draw a conclusion ! No men in a high position are free from envious regards, and many are in a high position.

10. " If the syllogism is shown to be valid by its own principles, what is the use of syllogistic rules ? " Explain.

11. Show how the general rules of the syllogism are derived from the " Dictum ".

12. State the rule concerning the Middle term. Why is it enough to distribute that term once ?

13. Prove that any extension of either term greater in the conclusion than in the premises involves a fallacy.

14. State and justify the syllogistic rules regarding (*a*) two negative premises, and (*b*) one negative premise.

15. Explain by general reasoning why two particular premises yield no conclusion. Is there any exception to this rule ? If so, give an example.

16. Prove by general principles that a valid syllogism with a particular premise must have a particular conclusion.

17. Point out the rule or rules which are broken in the following syllogisms :

(1) No inexperienced man is competent,
there are politicians not incompetent.
therefore some politicians are not experienced.

(2) Governments are good which promote prosperity,
the government of Russia does not promote prosperity,
therefore it is not a good government.

(3) The sea is visible from this place, and the hostel is visible from this place, therefore the hostel is visible from the sea.

(4) No tale-bearer is to be trusted, and therefore no great talker is to be trusted, for all tale-bearers are great talkers.

(5) No dishonest man is fit for a high position, the students of Inter-Arts are known to be honest, therefore those students are fit for a high position.

(6) Many unemployed people are not unskilful, all my friends are unemployed, therefore none of them are unskilful.

(7) Only unambiguous language is scientific, the language of Logic is unambiguous, therefore it must be scientific.

18. What are the syllogistic Figures ? Assuming the general rules of the syllogism, prove the particular rules of the Figures.

19. Why is it that the Second figure proves only negative, and the Third figure only particular conclusions ?

20. Show that an **O** proposition cannot be a premise in Fig. I., nor in Fig. IV.

21. Prove by the general rules that, wherever the minor premise is negative, the major premise must be universal.

22. State the total number of possible moods. By what way would you determine those which are valid ?

23. Assuming the particular rules of the figures, show that the Valid moods are six in each figure.

24. What are Strengthened and Weakened moods ? Explain why the former are admitted and the latter rejected in Logic.

25. In what figure can there be no Weakened moods ? Are the moods **AAI, EAO, AEO** weakened or strengthened ?

26. What is meant by Fundamental moods ? State the moods which are not fundamental and yet admitted.

27. Express the following arguments in syllogistic form, and, if valid, state figure and mood :

(1) Only members are admitted in this club, hence I conclude that Captain Smith is a member, for he is admitted.

(2) No dishonest persons can be trusted, hence some employees are unworthy of trust, for they are dishonest.
(3) All B is A, only C is A, therefore only C is B.
(4) All A is B, all not-C is not-B, therefore all A is C.
(5) No M is not-P, no S is not-M, therefore all S is P.
(6) Everything is either M or P, everything is either not-S or not-M, therefore everything is either P or not-S.
(7) With no attempt there can be no failure ; with no failure, no humiliation ; therefore with no attempt no humiliation.
(8) Everything is M or P, nothing is both S and M, therefore all S is P.
(9) What is incredible is not understood, some stories are not credible, therefore some stories are not understood.
(10) Some knowledge is not attainable without labour ; for some knowledge is valuable, and nothing which is attainable without labour is valuable.
(11) Most unemployed are not happy, and most unemployed are unskilful, therefore some unskilful men are not happy.
(12) If it is false that B is found co-existing with A, and not less false that C is sometimes absent from A, can you assert anything about B in terms of C ?
(13) The State ought to suppress evils ; betting is an evil ; therefore the State ought to suppress betting.
(14) It cannot be true that all repression is mischievous, if government is repressive and yet is sometimes beneficial.
(15) Although no graduates are ignorant, all are fallible ; hence not all who are fallible are ignorant.

28. Explain the characteristics of the four syllogistic figures. What is the use of the fourth figure ?

29. Show by reference to the valid moods that an **A** proposition is most difficult to prove and the easiest to disprove.

30. Build a syllogism in all four figures to prove that "some public expenses are not justifiable."

31. Assuming the general rules of the syllogism, prove :
(*a*) Whenever the minor term is distributed, the major premise must be universal.
(*b*) When the minor term is predicate in its premise, the conclusion cannot be an **A** proposition.
(*c*) If the conclusion of a syllogism is universal, the middle term can be distributed only once.

32. Given that an **O** proposition is major premise in a valid syllogism, find out the figure and mood.

33. What can be determined respecting a syllogism under each of the following conditions ?

(*a*) That the major term is distributed in the premises and undistributed in the conclusion.
(*b*) That only two terms are distributed, each only once.

34. Show by general reasoning that the contradictories of the premises of a valid syllogism will not in any case suffice to establish the contradictory of the original conclusion.

35. If the major premise and the conclusion differ from each other in quantity and quality, determine the mood and figure.

36. Why is **IEO** an inadmissible and **EIO** an admissible mood in every figure of the syllogism ?

37. Two valid syllogisms in the same figure have the same major, middle and minor terms, and their minor premises are sub-contraries. Determine the syllogisms.

38. What is meant by reduction of syllogisms ? Explain and illustrate (*a*) Direct, and (*b*) Indirect reduction.

39. State the purpose of reduction. How far is reduction (*a*) necessary or (*b*) useful in a course of logical doctrine ?

40. On what grounds would you defend the merits of the syllogism ? Point out (*a*) the usefulness, and (*b*) the limitations of the Syllogistic Form.

CHAPTER VIII

1. Consider the difficulties which confront the logician in the Analysis of Arguments.

2. How would you proceed in testing an argument ? Enumerate the kinds of arguments recognised in Formal Logic.

3. What is an Ethymeme ? And what are its varieties ? Fill out the following enthymemes, stating figure and mood.

(1) Bombay is an industrial city, and industrial cities are not health-resorts.
(2) You are not an engineer ; therefore you are not eligible for this post.
(3) All these people must be good citizens ; for only good citizens obey the law.
(4) Some statesmen are also authors ; for such are Gladstone, Balfour and others,
(5) Some eminent men have not obtained university distinction, which, therefore, is a non-conclusive proof of real ability.
(6) Only men of wide experience are tolerant ; hence it is that young people are usually intolerant.
(7) This proposal is " too good " to be practicable.
(8) Mercy but murders, " pardoning " those that kill.

(9) I do not derive my opinion from the newspapers ; for I never read any of them.

(10) Some men of self-command are poor, and therefore some noble characters are poor.

(11) We cannot infer honesty from the absence of intemperance, for Smith is neither honest nor intemperate.

4. Explain the Epichireme and its varieties. How does it differ from the Polysyllogism ?

5. Examine the validity of the following arguments :

(1) Classical learning deserves to be promoted, because classical learning, giving us a taste for intellectual enjoyments, withdraws the mind from pursuits of a low nature ; and whatever does this deserves to be promoted.

(2) Partners in the same enterprise have the same interest. How then can there be antagonism between my workmen and me ?

(3) He is a liar who says he loves God, and still hates mankind. For how can he love God whom he does not see, when he hates mankind whom he sees ?

6. Explain the forms of Aristotelian and Goclenian Sorites, and compare them with Polysyllogism.

7. State and prove the particular rules of the Sorites. Illustrate those rules with the following examples :

(1) Athletic games ought to be compulsory. For athletic games are exercise, exercise is necessary to health, things necessary to health ought to be compulsory ; therefore athletic games ought to be compulsory.

(2) Religion is not an exact science, an exact science is demonstrated, a demonstrated doctrine is true, what is true is profitable, therefore religion is not profitable.

(3) The educated man is not conceited, the conceited are rude, the rude hurt others' feelings, and in that they are not gentlemen, therefore the educated man is not a gentleman.

(4) Indians are politicians, some politicians are not patriots, all patriots love the country, and by loving the country, serve the country, therefore Indians serve the country.

(5) None but the contented are happy, none but the virtuous are contented, none but the wise are virtuous, therefore none but the wise are happy.

8. By what rules would you test a pure hypothetical syllogism ? Examine the following :

If there were no enemy, no fight ; if no fight, no victory ; therefore if no enemy, no victory.

9. Explain the mixed hypothetical syllogism, its valid and invalid moods.

10. What proof would you offer of the rules of valid inference in mixed hypothetical syllogisms ? Are these rules connected with the doctrine of plurality of causes ?

11. " In a mixed hypothetical syllogism the consequent may be denied by a contrary or by the contradictory, but it is essential to deny the antecedent by the contradictory." Explain.

12. Take the following example, and keeping the same minor premise and conclusion, transform its constructive mood into destructive :

> If any student has not kept terms, he cannot sit for examination; this student has not kept terms; therefore he cannot sit for examination.

13. Illustrate how mixed hypothetical syllogisms may be transformed into categorical, and point out the corresponding moods.

14. State the moods and the formal validity of the following mixed hypothetical arguments :

(1) If any one can square the circle, he is a great mathematician, but no one can.

(2) If the charge is false, the author of it is either ignorant or malicious ; but the charge is true ; therefore he is neither.

(3) Had all the students been prepared for the test, some would have succeeded ; but none have come out successful.

(4) We know that the policy was wrong ; for otherwise it would not have failed.

(5) Professional mistakes would only be excusable if they were unavoidable ; but they are avoidable.

(6) If some S is not-P, then no S is P ; but some S is P.

(7) If not-S is P, then some S is not-P, but some S is not-P.

(8) Party feeling must have been running high, or else the vote for the bad candidates would not have been so large.

(9) If philosophical theories are true, some at least ought to be accepted by a majority of thinkers ; but as not a single theory fulfils this condition, no philosophical theory can be true.

(10) If, and only if, a person is appointed as head-officer has he any authority over the employees. Mr. Brown is not appointed, therefore he has no authority over the employees.

15. What is a disjunctive syllogism ? Consider the conclusions which can be drawn from a disjunctive major premise.

16. State and justify the rules and conditions of valid inference in disjunctive syllogisms.

17. Transform the following disjunctive syllogism into (*a*) hypothetical, and (*b*) into categorical, and compare their respective moods :

> Either a man is capable of progress, or he does not differ from the brutes. But man does differ from the brutes.

18. Fill up, if necessary, the following disjunctive syllogisms and test their validity :

(1) He is either very good, very bad or commonplace. But he is not very good.
(2) A man so precise in walk and conversation must be either a saint or a consummate hypocrite. But, no doubt, he is a saint.
(3) A successful man must be either industrious or rich ; but this successful man is industrious ; therefore he is not rich.
(4) Either every S is P or no S is P. But some S is P.
(5) Either A is B or C is not D. But C is not D.
(6) John studies mathematics either from interest in the subject or in order to pass an examination, but he is studying it in order to pass an examination ; therefore he is not studying it from interest in the subject.
(7) Not both some S is P and some S is not P ; but this S is P.

19. Given : " If a man is honest, he acknowledges his mistakes ", draw a conclusion in the constructive mood ; and keeping the same minor premise and conclusion, change it into a disjunctive syllogism.

20. Describe (*a*) the structure of a dilemma, and (*b*) its moods of valid inference. Give examples.

21. State the rules for a valid dilemma, and consider the fallacies incident to this argument.

22. " A dilemma is most impressive in that we offer a choice of undesirable alternatives, but a missing alternative makes it openly invalid and thoroughly useless." Explain this with the following example : An employee is unsuccessful either on account of ill-health or as a result of financial depression. In either case he is not to blame.

23. Does rebuttal prove the invalidity of the rebutted dilemma ? Illustrate your answer with the following :

> If education is popular, compulsion is unnecessary ; if unpopular, compulsion will not be tolerated.

24. State fully and examine the validity of these dilemmas :

(1) Moral exhortations are useless. For good men don't need them, and bad men will pay no heed to them.
(2) Either a slave is capable of virtue, or he is not; therefore he ought not to be a slave, or he is not a man.
(3) Never speak well or ill of yourself. If well, men will not believe you ; if ill, they will believe a great deal more than you say.
(4) A poor man is either healthy or unhealthy. Therefore he must either work daily or suffer misery.

25. " If that man were wise, he would not speak against the sacred books in jest; and if he were good, he would not do so in earnest. But he does it either in jest or in earnest ; therefore he is either not wise or not good." Reverse this dilemma from the destructive to the constructive form.

26. Does the syllogism cover all forms of deductive reasonings ? Discuss.

27. By what principles would you ascertain the validity of relational arguments ? Examine the following :

(1) A is taller than B ; B is equal in height to C ; therefore A is taller than C.
(2) A is west of B ; B is west of C ; therefore A is west of C.
(3) A is the creditor of B ; B is the creditor of C ; therefore A is the creditor of C.

28. Consider the nature of probable deductive arguments. Analyse the following :

(1) The governor is either in Poona or in Bombay ; but he is not likely to be in Poona ; therefore he is more probably in Bombay.
(2) Writings of similar style are probably from the same author ; these writings are similar in style ; therefore they are probably from the same author.
(3) A substance may not be gold, although it glitters ; this substance glitters ; therefore it may not be gold.
(4) There are many employees who are not successful, and most employees are honest ; therefore some honest employees are not successful.

29. What is a Deductive Fallacy ? Enumerate briefly the main forms of deductive fallacies.

30. Ascertain the character of the following arguments, indicating any fallacy they may contain :

(1) Some people are not good citizens ; for all who obey the law are good citizens, and some people do not obey the law.

(2) Only trespassers are liable to prosecution ; this man is a trespasser ; therefore he is liable to prosecution.

(3) Warm countries alone produce wine ; Spain is a warm country ; therefore Spain produces wine.

(4) Some politicians are selfish ; and some selfish men betray their trust ; and none who betrays his trust is fit for his duty.

(5) The atmosphere cannot be a conductor of electricity ; for metals are conductors of electricity, and the atmosphere is not a metal.

(6) If all educated men were true to their duties, the country would be very prosperous. But it is not very prosperous ; therefore the educated are not true to their duties.

(7) None but truthful men are honest ; none but truthful men are worthy of respect, therefore all men who are worthy of respect are honest.

(8) He must be either ignorant or a liar ; he is ignorant ; therefore he cannot be a liar.

(9) Any man who succeeds in life must be either industrious or rich ; but many people are neither industrious nor rich ; therefore many people cannot succeed in life.

(10) A, B and C are P ; all S is either A or B or C ; therefore all S is P.

(11) If any steamer arrives at low tide, the passengers must either land in small boats, or wait several hours. But they cannot wait several hours ; therefore if any steamer arrives at low tide the passengers must land in small boats.

(12) All who attacked the fort were killed or wounded ; no cowards attacked the fort ; therefore none who were killed or wounded were cowards.

(13) You cannot affirm that all his acts were virtuous, for you deny that they were all praiseworthy, and you allow that nothing that is not praiseworthy is virtuous.

CHAPTERS IX and X

1. Explain the problem which Inductive Logic intends to solve, and the situation that gives rise to the problem.

2. Compare the process of Induction with that of Deduction. Do they differ in kind or only in the method of thinking ?

3. What is induction by Simple Enumeration ? Give examples of the so-called inductive syllogism which concludes by complete and incomplete enumeration.

4. Inquire into the claim of Complete Enumeration to the title of Perfect Induction.

5. What is meant by Scientific Induction? And how does it differ from Simple Enumeration?

6. "The extension of general names is not exhaustively known. What right have we then to state universal propositions concerning matters of fact?" Consider this question in its bearing on induction.

7. Under what condition can a universal conclusion be legitimately drawn from a single instance?

8. Outline the main steps leading to an inductive generalization.

9. State the presuppositions of induction, and the part they play in inductive inference.

10. Distinguish Unity from Uniformity of nature. How is it that we come to believe in some uniformities more than in others?

11. Inquire into the possibility of regarding the uniformity of nature as an induction from experience. Can it be proved?

12. State the Law of Universal Causation, and the axioms which, from the standpoint of Science, are implied in it.

13. Distinguish between the popular and the scientific notion of a cause. Are we justified in supposing that cause and effect are (*a*) equal, and (*b*) reciprocal?

14. Give an account of the empirical view of the scientific cause.

15. Illustrate the meaning of Complexity of causes, Intermixture of effects and Plurality of causes.

16. "If we knew the facts minutely enough it would be found that there is only one cause for each effect." Explain this, and why we are nevertheless practically bound to recognise a plurality of causes.

17. Consider how far the doctrine of plurality of causes affects the inference in hypothetical syllogisms.

18. Comment on each of the following statements:

(1) Induction by simple enumeration depends on the number of instances.
(2) Induction is inverse deduction.
(3) Inductive inference is no less formal than material.
(4) The course of nature is at once uniform and infinitely varied.
(5) The relation between cause and effect implies uniformity.
(6) There are causal and casual events, and likewise there are sequence and consequence among naturally occurring phenomena.

19. Examine the logical character of the following inductive inference.

    (1) Earthquakes have occurred from time to time in California ; therefore they will similarly happen in the future.
    (2) Looking at the islands marked down in a map of Europe, I came to the conclusion that none is very large.
    (3) All the great empires that ever existed have lost their position of eminence ; hence no great empire in the future will maintain its supremacy.

20. Illustrate with the following examples the precise meaning of symptom, condition, cause and reason :

    (1) One swallow does not make a summer, but some time before summer comes the first swallow has to appear.
    (2) Automatic signs should not be allowed along the highways, because they divert the driver's attention and thus constitute a traffic menace.
    (3) Tuberculosis is caused by the growth of a microscopic germ in the lungs and other parts of the body.
    (4) I set off on a journey, and the motive was to visit my mother.

CHAPTERS XI and XII

1. What is a Fact ? In what sense are facts the foundation of science ?

2. Explain the nature of Scientific Observation and how it tends to be Selective.

3. Illustrate the sources of error in observation.

4. Explain the use of instruments as an aid to observation. State the qualities required in the observer.

5. What is an Experiment ? And how does it differ from an Observation ?

6. " A perfect experiment establishes a law." Explain this and point out (*a*) the difficulties, and (*b*) the advantages of experiment from the standpoint of different sciences.

7. Can non-observation ever be a proof of the non-existence of an event ? Examine this case :

> A man accused of theft on the evidence of two witnesses offered to bring in in his defence ten witnesses who had not seen him commit the theft.

8. Explain the conditions of truth (*a*) in Oral Testimony, and (*b*) in Historical Documents.

9. State the place and function of Hypotheses in induction.

10. What do you take to be the conditions of a legitimate hypothesis ? Describe the various kinds of hypotheses.

11. Explain the Direct and the Indirect process of establishing an hypothesis. Give examples.

12. Given a verifiable hypothesis. What is it that constitutes its proof or disproof ?

13. Compare and illustrate the respective meaning of the terms Fact, Theory, Law, Hypothesis and Generalization.

14. State the various kinds of laws in Science. How are empirical laws distinguished from laws of nature ? Apply your distinction to the following statements :

(1) The barometer falls as one goes up a mountain.
(2) All cases of cancer are incurable.
(3) All breathing animals are hot-blooded.
(4) All peninsulas in the northern hemisphere point southwards.

15. What is meant by Scientific Explanation ? State the main modes of explanation in Science.

16. Compare Scientific Explanation (*a*) with Induction, (*b*) with Deduction, and (*c*) with Description.

17. What is a System of Knowledge ? Show the respective functions of induction and deduction in Science.

CHAPTERS XIII and XIV

1. What are the Inductive Methods ? And how are they connected with the hypotheses ?

2. Illustrate the analytic process followed in inductive methods, and show that their common aim is to accomplish Elimination.

3. Show that the principles by which the methods of induction proceed are derived from the Law of Causation.

4. Is a method of induction by itself a real proof of the existence of a cause ? If not, state the premises assumed in inductive inference.

5. Describe the Method of Agreement, and consider how far it fails in the analysis of facts.

6. Discuss the value of the Method of Difference. Is it of itself sufficient to logically establish a causal relation ?

7. Explain the Joint Method of Agreement and Difference. When is this method particularly needed ?

8. What is the Method of Concomitant Variations ? Consider its value for establishing quantitative relations.

9. State the use of the Method of Residues, and show that it is rather a hypothesis than a method.

10. Take an inductive generalization by any of the methods and set out at full length the reasoning and assumptions involved in it.

11. Consider to what extent plurality of causes affects the methods of Agreement and Difference.

12. Which method, in your opinion, affords the strongest and which the weakest ground for establishing a causal relation ?

13. "Mill's Methods have discovered nothing. Originality in thinking is what makes for real discovery." Discuss.

14. Explain the nature and scope of the Deductive Inductive Method.

15. Describe the Physical Method and the Historical Method. State the kind of phenomena to which each process is most appropriate.

16. "Mathematical demonstration is deductive, but any original work in Mathematics is an inductive process." Discuss.

17. Consider the practical use of the Statistical Induction, and compare it with Simple Enumeration, and with the Method of Agreement.

18. Distinguish proper from improper, and perfect from imperfect induction, and classify accordingly the various kinds of induction.

19. Given A B C as antecedents of X, how would you proceed to discover the precise cause of X ?

20. Draw the inferences deducible from these data :

(1) A is the only antecedent always present when *p* is present, and always absent when *p* is absent.

(2) A is an antecedent always present when *p* is present, and always absent when *p* is absent.

(3) A is an antecedent frequently present when *p* is present, and frequently absent when *p* is absent. (Park.)

21. A student of geometry examines three isosceles triangles and finds that they agree in having equal angles at the base ; an excise officer examines three bottles of wine out of a quantity imported, and finds them agreeing in strength ; a chemist analyses three specimens of a mineral and finds all agree in composition : compare the inferences which may be drawn in these cases. (Jevons.)

22. The question being whether A or B is probably the writer of an anonymous letter, by what kind of inductive inference might a decision on this point be reached ?

23. State the method and the principle implied in each of the following cases :

(1) Pasteur disproved spontaneous generation by two instances of equal fermentable substances, one in a closed air-tight vessel, the other exposed, both under

identical conditions. The result, positive and negative, corresponded to the exposure and non-exposure, thus showing that the germs had been imported through the air.

(2) "Brewster accidentally took an impression from a piece of mother-of-pearl in a cement of resin and bees' wax, and finding the colours repeated upon the surface of the wax, proceeded to take other impressions in balsam, fusible metal, lead, gum-arabic, etc., and always found the iridescent colours the same. He thus proved that the chemical nature is wholly a matter of indifference, and the form of the surface is the condition of such colours." (Jevons.)

(3) A patient whose illness had been traced to the proper cause, was gradually getting better, when a spell of fever reappeared. This sudden change, the doctor said, must be from a different cause, and he concluded that the patient had taken furtively some extra food.

(4) Several industrial plants in different countries have gone from the ten-hour day to the eight-hour day without change in the employees or in the wages. After a year all the companies declared that their financial conditions had grown better. It follows, therefore, as a general rule, that a similar change in the hours of work will benefit all kinds of industrial enterprise.

(5) Many of the pine trees in certain districts grow with twisted instead of straight fibre. In order to discover whether the twisting of the fibre is hereditary or due to the place in which the trees grow, the seed of the twisted tree is taken and planted in a number of different districts. On what principle is this experiment based? Is it likely to lead to a sure conclusion?

(6) We observe frequently that poor handwriting characterises the manuscripts of able men, while good handwriting is common among those who do little mental work. We may, therefore, infer that poor handwriting is caused by the influence of severe mental occupation.

24. Examine logically the following passage:

In a Bombay hospital a popular remedy other than quinine was tried in malaria cases. "The patients received no treatment beyond the usual hospital care, until the blood examination had been completed and the type of the disease determined. The treatment was then commenced and results noted, side by side

> with a daily qualitative and quantitative examination of blood films for malaria parasites. The disappearance of fever and the absence of parasites extending over a period of two weeks was taken as an indication of the beneficial effect of the drug." (Dr. Mhaskar, Haffkine Institute.)

25. Make out in the following passage the problem the hypothesis, the method and the force of the argument :

> " It is indeed desirable that youth be surrounded with a certain amount of comfort. But sufficient wage, good housing conditions, State-supervision, recreation and education cannot fill the gap left open by the lack of spiritual guidance which has for its purpose respect for constituted law and reverence for God, the fountain of all law, together with the inculcating of the idea of personal responsibility to the laws of God and man.
>
> " I have in mind the cases of two boys ; one was the product of the streets. He was short-changed, as it were, in many ways. He had a background of bad heredity, incompetent and mentally defective parents, poor education, out of work during the depression, and living in a crowded part of the city. The other seems to have had everything—good parents, good home, with plenty of material wealth, good school environment, good companions, and living in a rather exclusive suburb out in the open spaces. Both committed murder. I wonder if these boys, the one seventeen years of age, the other sixteen, had really been taught self-control and obedience to authority, with certain punishment for violating that authority.
>
> " Of course, to the State must be given a share of the blame for the surroundings and conditions in which it permits its boys to live ; the home may be at fault and the school may have misunderstood. But the real trouble has been and is the gradual drifting away from that great power that trains the soul and keeps ever keen the edge of conscience: that is, the regular practising of one's Faith." (Judge, Superior Court.)

26. What method would you consider most appropriate to discover :

(1) The cause of an outbreak of cholera in a large city.
(2) The conditions responsible for the sudden decline of a banking institution.

(3) The source of a headache which a person happens to feel after an intense reading.
(4) The amount of literacy or illiteracy among the people in a province.
(5) The cause or causes which brought about the great war.

CHAPTER XV

1. Explain the relation in which Probability stands to induction.

2. Examine the grounds of logical probability. In what sense are these grounds the foundation of our belief ?

3. State the rule to estimate the probability concerning (*a*) simple events, (*b*) the occurrence of either one or the other of two events, (*c*) the concurrence of two events.

4. What is a Mean Observation ? Consider the assumptions by which we can obtain the most probable measurement.

5. Point out the rule to estimate the probability of a conclusion from (*a*) Self-corroborative arguments, and (*b*) Self-informative arguments.

6. Explain the nature and value of inference from Analogy. What is the place of Analogy in induction, and how does it differ from Statistics ?

7. Examine the logical value of the following reasonings :

(1) Many people in Bombay suffer occasionally from fever ; hence animals may likewise suffer.
(2) We do not dispense with clocks because, from time to time they go wrong, and tell untruly ; likewise a servant should not be put out of service because sometimes he makes mistakes.
(3) " Associations like joint-stock companies are best managed by a committee chosen from among themselves. Hence the best form of national government is by a popularly elected assembly." (Westaway.)
(4) " Counterfeit coin supposes that there is such a thing in the world as good money, and no one would pretend outwardly to be virtuous unless some really were so. In the same manner false miracles suppose the existence of real ones, and the cheats that have been imposed upon the world, far from furnishing us with reasons for rejecting all miracles in general, are, on the contrary, a strong proof that some, of which they are imitations, have been genuine." (Latta and Macbeath.)

8. The testimony being taken of three witnesses independently of each other, all agreed in the guilt of the accused.

Estimate the joint probability of the three testimonials on the supposition that none was absolutely certain.

9. Concerning a family-theft someone reasoned this way : it is probable that the culprit is one of the household, very likely one of the servants, and probably the youngest of them in the service. Estimate logically the probability of the conclusion.

CHAPTER XVI

1. State the purpose of Classification, and its relation to induction.
2. Explain the process of classification. What are the sciences which largely depend on classification ?
3. Distinguish between Natural and Artificial classification. Give examples.
4. Consider the respective value of classification by Type and by Kinds.
5. Give an account of classification as modified from the point of view of Evolution.
6. State the underlying principles of classification Why is a natural classification difficult and liable to revision ?
7. Explain the relation of logical Division to Classification.
8. What is meant by Nomenclature and Terminology ? State the reasons why scientific classification should be expressed in technical terms.
9. " Most names current in ordinary language are difficult to define, while scientific ones are capable of sharp definitions." Explain this with reference to classification.

CHAPTER XVII

1. Give an account of the main sources of fallacies.
2. Explain the terms Fallacy, Error, Paralogism, and Sophism. Attempt a general division of fallacies.
3. Explain the nature of Formal and Semiformal fallacies. Illustrate the distinction between the two.
4. What is a Material fallacy ? Show that it is the matter and not the form that leads to a wrong conclusion.
5. What is meant by Begging the Question ? Point out the prevailing modes of this fallacy in the course of argument.
6. On what grounds has it been maintained that the syllogism begs the question ? Discuss.
7. Explain the fallacy of Evading the Question. Consider the use this fallacy is put to, for bringing about persuasion.
8. How would you define an Inductive fallacy ? Illustrate False Cause and False Generalisation.

9. Consider to what extent the knowledge of fallacies may contribute to the cause of truth.

10. Name and explain the kind of fallacy, whether semi-formal or material, in each of the following reasonings :

(1) To kill a man is a crime ; a murderer is a man ; therefore to kill a murderer is a crime.

(2) No news is good news ; I have brought you no news ; hence I have brought you good news.

(3) To be wealthy is not to be healthy ; not to be healthy is to be miserable ; therefore to be wealthy is to be miserable.

(4) One number of the lottery is sure to win the prize ; my ticket is one number of the lottery ; therefore my ticket is sure to win the prize.

(5) What we eat grew in the fields ; loaves of bread are what we eat ; therefore loaves of bread grew in the fields.

(6) I can afford to buy these books ; I can afford to buy these pictures and these silver-pots too. The books, the pictures and the silver-pots are all that I, at present, wish to buy. I can, therefore, buy everything I want to buy.

(7) There is not a single branch of knowledge that can be said to be indispensable ; therefore a man may dispense with knowledge altogether.

(8) It is true that in democracy the people vote, but it is absurd to say that they rule, for you and I are people, and everything we vote for is defeated.

(9) The spread of education among the lower classes will make them unfit for their work, for such has been the case among the menials in our factory.

(10) The proportion of inmates in our asylums who can read and write is very high, from which we may infer that education is among the causes of insanity.

(11) Whatever moves around us catches the attention. Therefore we are more likely to notice a moving object than one that remains stationary.

(12) The bill before the house is well calculated to improve the character of education. for the education will be raised in all the schools by the proposed bill.

(13) I cannot accept your opinion as true, for it seems to me that its general recognition would be attended with serious consequences to society.

(14) What would our ancestors say to this bill ? How does it agree with their experience ? Are we to put the wisdom of yesterday in competition with the wisdom of centuries ?

(15) It is easy to cast stones at this great man; but it would be more rational to ask how many of us would have come through such a career of temptation with a better record.

(16) The love of money is the root of all evil; we must not therefore entertain, on any account, applications for promotions from our employees.

(17) Free trade is suited to England; therefore it is suited to India, which is part of the same empire.

(18) Knowledge of Logic is not useful, because it does not teach matters of business.

(19) Moral and religious instruction can be of no use whatever, since we know from our experience of men that a great many rogues have a considerable knowledge of religion and morals.

(20) "Suppose I am walking out in the moonlight, and see dimly the outlines of some figure among the trees —it is a man. I draw nearer,—it is still a man; nearer still, and all hesitation is at an end,—I am certain it is a man. But he neither moves, nor speaks when I address him; and then I ask myself what can be his purpose in hiding among the trees at such an hour. I come quite close to him, and put out my arm. Then I find for certain that what I took for a man is but a singular shadow, formed by the falling of the moonlight on the interstices of some branches or their foliage. Am I not to indulge my second certitude, because I was wrong in my first? Does not any objection, which lies against my second from the failure of my first, fade away before the evidence on which my second is founded?" (Newman.)

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